Assignment 1
Communication Theory EE304

Submit Qts.: 4, 7, 10(a, b, e), 12, 14, 16 and 17.

1. Find the energies of the signals shown in Fig. 1. Comment on the effect on energy of sign change, time shifting or doubling of the signal. What is the effect on the energy if the signal is multiplied by k?

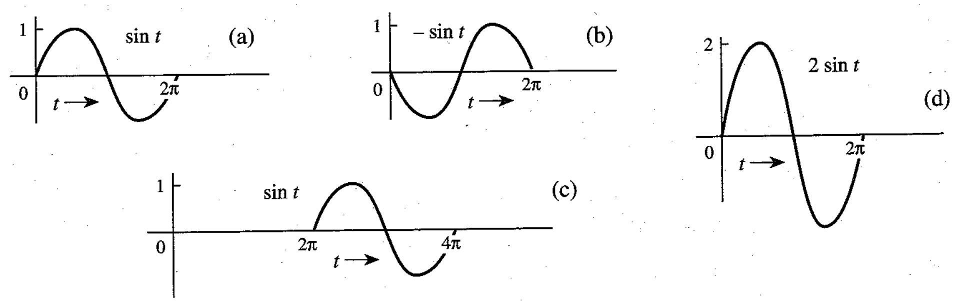

Figure 1:

2. (a) Find E_x and E_y, the energies of the signals x(t) and y(t) shown in Fig. 2a. Sketch the signals $x(t) + y(t)$ and $x(t) = y(t)$ and show that the energies of either of these two signals are equal to $E_x + E_y$. Repeat the procedure for the signal pair of Fig. 2b.
   (b) (b) Repeat the procedure for the signal pair of Fig. 2c. Are the energies of the signals $x(t) + y(t)$ and $x(t) - y(t)$ identical in this case?
3. (a) Determine the power and the rms value (Signal power is square of its rms value) of $g(t) = Ccos(\omega_0 t + \theta)$
   (b) Find the power of a sinusoid $Ccos(\omega_0 t + \theta)$ by averaging the signal energy over one period $2\pi/\omega_0$ (rather than averaging over the infinitely large interval).
4. Find the power of the periodic signal $g(t)$ shown in Fig. 3. Find also the powers and the rms values of:
   (a) -g(t)
   (b) 2g(t)
   (c) cg(t)
   (d) Comment.

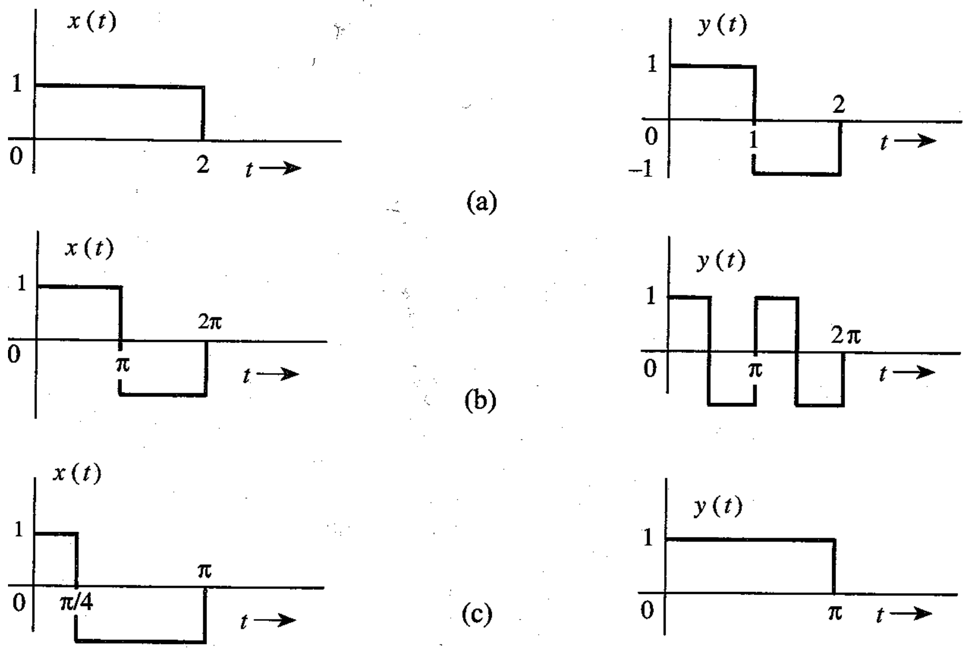

Figure 2:

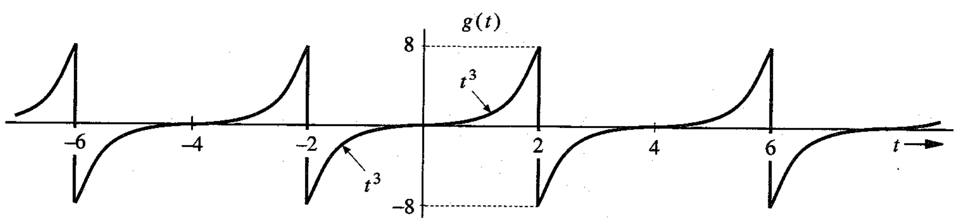

Figure 3:

5. Show that an exponential e^{-at} starting at $-\infty$ is neither an energy nor a power signal for any real value of a. However, if a is imaginary, it is a power signal with power $P_g = 1$ regardless of the value of a.

6. For the signals $g(t)$ and $x(t)$ shown in Fig. 4, find the component of the form $x(t)$ contained in $g(t)$. In other words, find the optimum value of c in the approximation $g(t) \approx cx(t)$ so that the error signal energy is minimum. What is the error signal energy?

Figure 4:

7. Energies of the two energy signals $x(t)$ and $y(t)$ are E_x and E_y, respectively.

(a) If $x(t)$ and $y(t)$ are orthogonal, then show that the energy of the signal $x(t) + y(t)$ is identical to the energy of the signal $x(t) - y(t)$, and is given by $E_x + E_y$.

(b) If $x(t)$ and $y(t)$ are orthogonal, find the energies of signals $c_1x(t)+c_2y(t)$ and $c_1x(t) - c_2y(t)$.

(c) We define E_{xy}, the cross energy of the two energy signals $x(t)$ and $y(t)$, as

$$E_{xy} = \int_{-\infty}^{+\infty} x(t)y^{\star}(t)dt$$

If $z(t) = x(t) \pm y(t)$, then show that $E_z = E_x + E_y \pm (E_{xy} + E_{yx})$

8. If a periodic signal satisfies certain symmetry conditions, the evaluation of the Fourier series components is somewhat simplified. Show that:

   (a) If $g(t) = g(-t)$ (even symmetry), then all the sine terms in the Fourier series vanish ($b_n = 0$).

   (b) If $g(t) = -g(-t)$ (odd symmetry), then the dc and all the cosine terms in the Fourier series vanish ($a_0 = a_n = 0$).

   Further, show that in each case the Fourier coefficients can be evaluated by integrating the periodic signal over the half-cycle only. This is because the entire information of one cycle is implicit in a half-cycle due to symmetry. *Hint:* If $g_e(t)$ and $g_o(t)$ are even and odd functions, respectively, of t, then (assuming no impulse or its derivative at the origin)

$$\int_{-a}^{a} g_e(t)dt = 2\int_{0}^{a} g_e(t)dt \quad and \quad \int_{-a}^{a} g_o(t)dt = 0$$

   Also the product of an even and an odd function is an odd function, the product of two odd functions is an even function, and the product of two even functions is an even function.

9. For each of the periodic signals shown in Fig. 5, find the compact trigonometric Fourier series and sketch the amplitude and phase spectra. If either the sine or the cosine terms are absent in the Fourier series, explain why.

10. For each of the periodic signals in Fig. 5, find exponential Fourier series and sketch the corresponding spectra.

11. A periodic signal $g(t)$ is expressed by the following Fourier series: $g(t) = 3cost + cos(5t - \frac{2\pi}{3}) + 2cos(8t + \frac{2\pi}{3})$

   (a) Sketch the amplitude and phase spectra for the trigonometric series.

   (b) By inspection of spectra in part (a), sketch the exponential Fourier series spectra.

   (c) By inspection of spectra in part (b), write the exponential Fourier series for $g(t)$.

12. Show that the Fourier transform of $g(t)$ may be expressed as

$$G(\omega) = \int_{-\infty}^{\infty} g(t)cos\omega t\ dt - j\int_{-\infty}^{\infty} g(t)sin\omega t\ dt$$

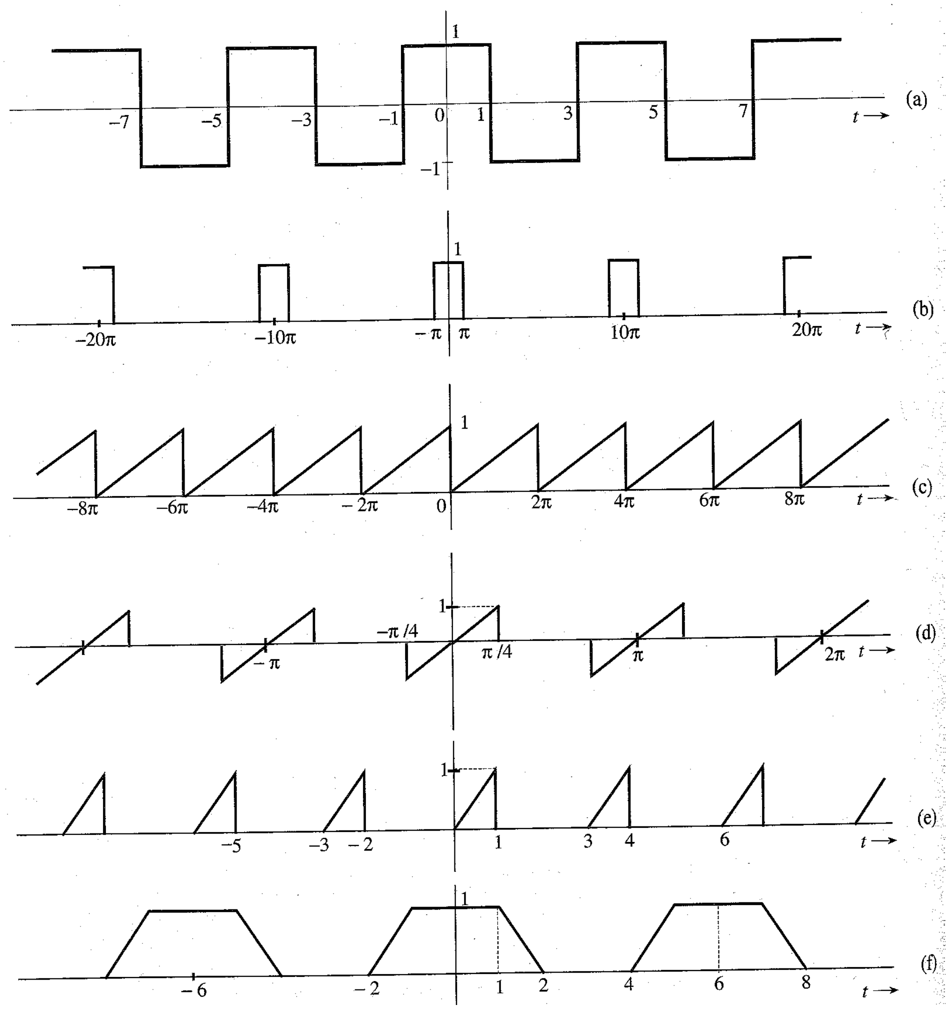

Figure 5:

Hence, show that if $g(t)$ is an even function of t, then

$$G(\omega) = 2\int_0^{\infty} g(t)cos\omega t\ dt$$

and if $g(t)$ is an odd function of t, then

$$G(\omega) = -2j\int_0^{\infty} g(t)sin\omega t\ dt$$

Hence, prove that:

If $g(t)$ is:	Then $G(\omega)$ is:
a real and even function of t	a real and even function of ω
a real and odd function of t	an imaginary and odd function of ω
an imaginary and even function of t	an imaginary and even function of ω
a complex and even function of t	a complex and even function of ω
a complex and odd function of t	a complex and odd function of ω

13. Prove the following results:

$$g(t)\sin\omega t \Longleftrightarrow \frac{1}{2j}[G(\omega-\omega_o)-G(\omega+\omega_0)]$$
$$\frac{1}{2j}[g(t+T)-g(t-T)] \Longleftrightarrow G(\omega)\sin T\omega$$

Using the later result and standard results, find the Fourier transform of the signal in Fig. 6.

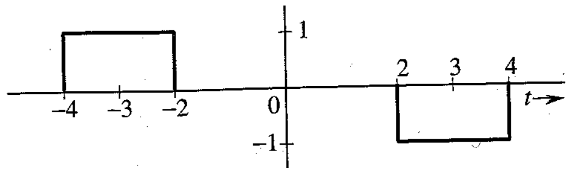

Figure 6:

14. The signals in Fig. 7 are modulated signals with carrier $\cos 10t$. Find the Fourier transforms of these signals using the appropriate properties of the Fourier transform and standard results. Sketch the amplitude and phase spectra for parts (a) and (b). Hint: These functions can be expressed in the form $g(t)\cos\omega t$.

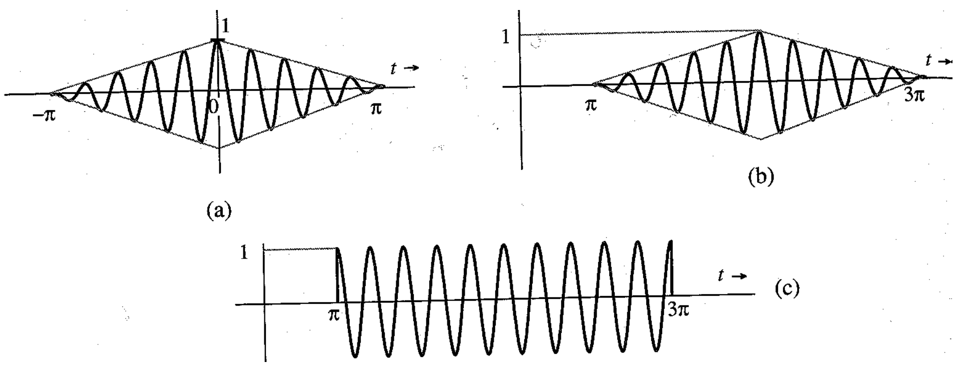

Figure 7:

15. Suppose that a set of M signal waveforms $s_{lm}(t$ is complex-valued. Derive the equations for the Gram-Schmidt procedure that will result in a set of $A < M$ orthonormal signal waveforms.

16. Consider the three waveforms $f_n(t)$ shown in Fig. 8.

    (a) Show that these waveforms are orthonormal.

    (b) Express the waveform $x(t)$ as a linear combination of $f_n(t), n = 1, 2, 3$, if

    $$x(t) = \begin{cases} -1 & (0 \le t \le 1) \\ 1 & (1 \le t \le 3) \\ -1 & (3 \le t \le 4) \end{cases}$$

    and determine the weighting coefficients.

17. Consider the four waveforms shown in Fig. 9.

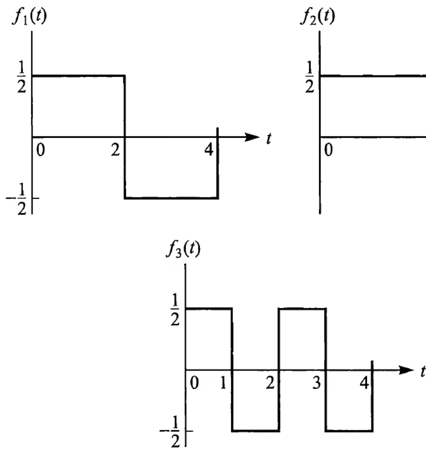

Figure 8:

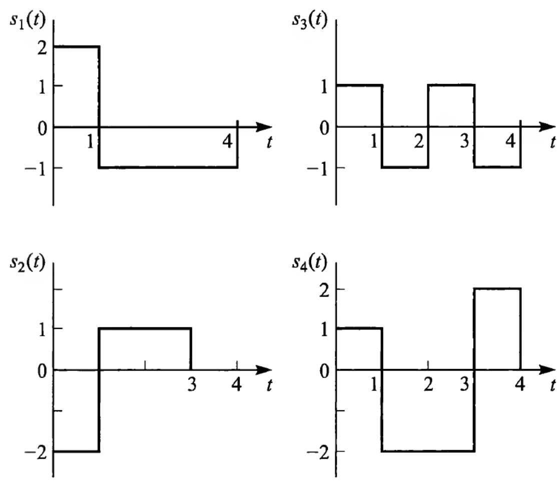

Figure 9:

(a) Determine the dimensionality of the waveforms and a set of basis functions.

(b) Use the basis functions to represent the four waveforms by vectors s_1, s_2, s_3 , and s_4.

(c) Determine the minimum distance between any pair of vectors.

18. Determine a set of orthonormal functions for the four signals shown in Figure 10.

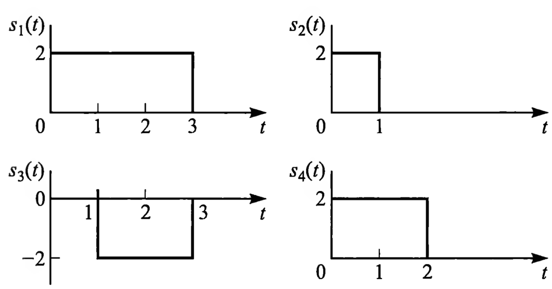

Figure 10: