Assignment 2
Communication Theory EE304

Submit Qts.: 2, 4, 6, 11, 12 and 13.

1. For each of the baseband signals,

   (a) $m(t) = \cos 1000t$;

   (b) $m(t) = 2\cos 1000t + \sin 2000t$;

   (c) $m(t) = \cos 1000t \cos 3000t$;

   (d) $m(t) = \text{sinc}(100t)$;

   (e) $m(t) = e^{-|t|}$ and $m(t) = e^{-|t-1|}$ (Observe that $e^{-|t-1|}$ is $e^{-|t|}$ delayed by 1 second.);

   (f) $m(t) = e^{-|t|}$ if the carrier is $\cos(10,000t - \pi/4)$;

   do the following.

   i. Sketch the spectrum of $m(t)$.

   ii. Sketch the spectrum of the DSB-SC signal $m(t)\cos 10,000\pi t$.

   iii. Identify the upper sideband (USB) and the lower sideband (LSB) spectra.

   iv. For (i)-(iii), identify the frequencies in the baseband, and the corresponding frequencies in the DSB-SC, USB, and LSB spectra. Explain the nature of frequency shifting in each case.

2. Design a DSB-SC modulator to generate a modulated signal $km(t)\cos(\omega_c t + \theta)$, where $m(t)$ is a signal band-limited to B Hz. Figure 1 shows a DSB-SC modulator available in the stock room. The carrier generator available generates not $\cos \omega_c t$, but $\cos^3 \omega_c t$, Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.

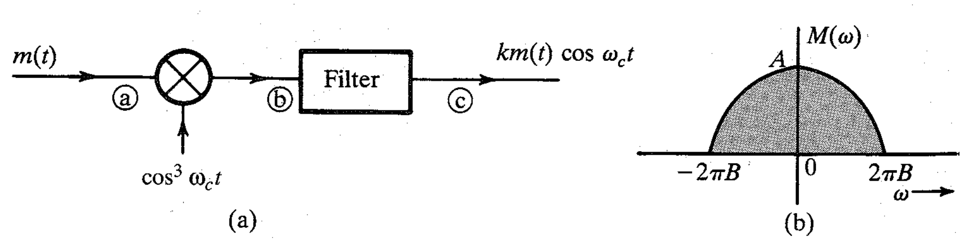

Figure 1:

   (a) What kind of filter is required in Fig. 1?

(b) Determine the signal spectra at points b and c, and indicate the frequency bands occupied by these spectra.

(c) What is the minimum usable value of we?

(d) Would this scheme work if the carrier generator output were $\cos^2 \omega_c t$? Explain.

(e) Would this scheme work if the carrier generator output were $\cos^n \omega_c t$ for any integer $n \geq 2$?

3. Amplitude modulators and demodulators can also be built without using multipliers. In Fig. 2, the input $\phi(t) = m(t)$, and the amplitude $A >> |\phi(t)|$. The two diodes are identical, with a resistance of r ohms in the conducting mode and infinite resistance in the cutoff mode. Show that the output $e_0(t)$ is given by $e_0(t) = \frac{2R}{R+r} w(t)m(t)$, where $w(t)$ is the switching periodic square wave signal (unit amplitude between $(-\frac{\pi}{2}, \frac{\pi}{2})$ and zero elsewhere) with period $2\pi/\omega_c$ seconds.

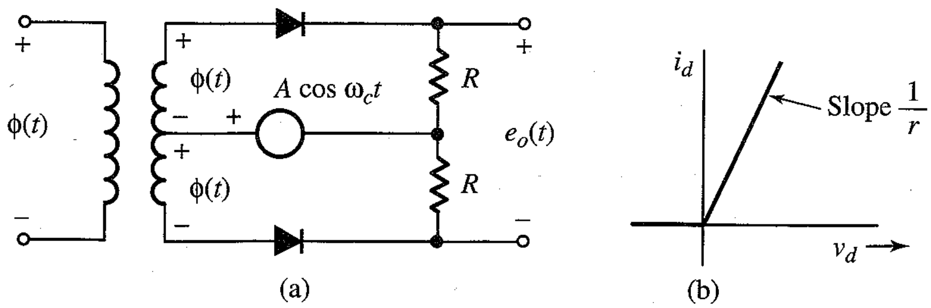

Figure 2:

(a) Hence, show that this circuit can be used as a DSB-SC modulator.

(b) How would you use this circuit as a synchronous demodulator for DSB-SC signals.

(c) In Fig. 2, if $\phi(t) = \sin(\omega_c t + \theta)$, and the output $e_0(t)$ is passed through a low-pass filter, then show that this circuit can be used as a phase detector, that is, a circuit that measures the phase difference between two sinusoids of the same frequency (ω_c). Hint: Show that the filter output is a dc signal proportional to $\sin\theta$.

4. Two signals $m_1(t)$ and $m_2(t)$, both band-limited to 5000 rad/s, are to be transmitted simultaneously over a channel by the multiplexing scheme shown in Fig. 3. The signal at point b is the multiplexed signal, which now modulates a carrier of frequency $20,000$ rad/s. The modulated signal at point c is transmitted over a channel.

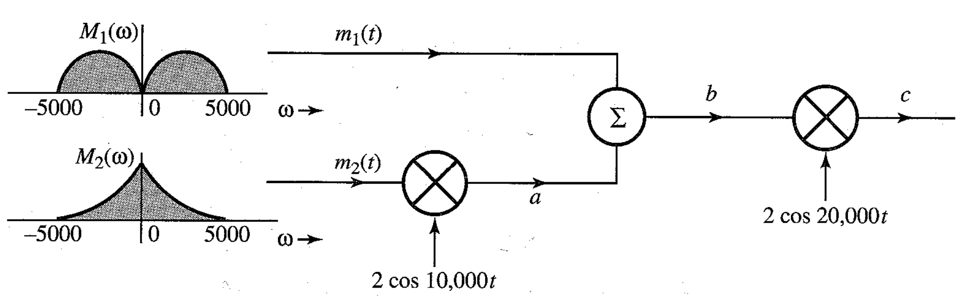

Figure 3:

(a) Sketch signal spectra at points a, b, and c.

(b) What must be the bandwidth of the channel?

(c) Design a receiver to recover signals $m_1(t)$ and $m_2(t)$ from the modulated signal at point c.

5. Figure 4 shows a scheme for coherent (synchronous) demodulation. Show that this scheme can demodulate the AM signal $[A + m(t)]\cos(\omega_c t)$ regardless of the value of A.

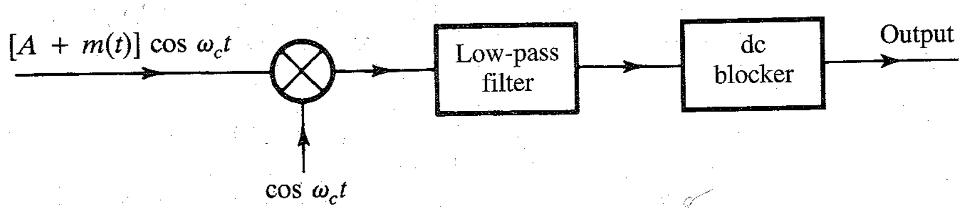

Figure 4:

6. Sketch the AM signal $[A + m(t)]\cos(\omega_c t)$ for the periodic triangle signal $m(t)$ shown in Fig. 5 corresponding to the modulation indices,

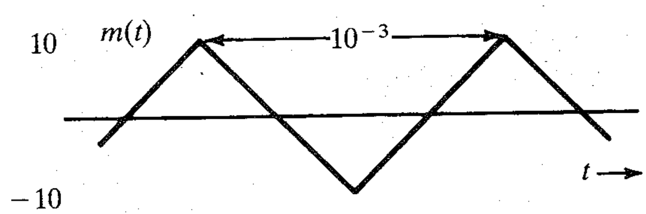

Figure 5:

(a) $\mu = 0.5$.

(b) $\mu = 1$.

(c) $\mu = 2$.

(d) $\mu = \infty$. How do you interpret the case of $\mu = \infty$?

(e) For the AM signal with $m(t)$ and $\mu = 0.8$. Find,

i. the amplitude and power of the carrier.

ii. the sideband power and the power efficiency η.

7. Show the following,

(a) Any scheme that can be used to generate DSB-SC can also generate AM. Is the converse true? Explain.

(b) Any scheme that can be used to demodulate DSB-SC can also demodulate AM. Is the converse true? Explain.

8. In the early days of radio, AM signals were demodulated by a crystal detector followed by a low-pass filter and a de blocker, as shown in Fig. 6. Assume a crystal detector to be basically a squaring device. Determine the signals at points a, b, c, and d. Point out the distortion term in the output $y(t)$. Show that if $A >> |m(t)|$, the distortion is small.

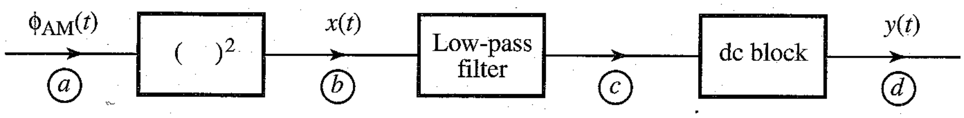

Figure 6:

9. A modulating signal m(t) is given by:
   (a) $m(t) = \cos 100t$
   (b) $m(t) = \cos 100t + 2\cos 300t$
   (c) $m(t) = \sin 100t \sin 500t$
   In each case:
      i. Sketch the spectrum of $m(t)$.
      ii. Find and sketch the spectrum of the DSB-SC signal $2m(t)\cos 1000t$.
      iii. From the spectrum obtained in (ii), suppress the LSB spectrum to obtain the USB spectrum.
      iv. Knowing the USB spectrum in (ii), write the expression $\phi_{USB}(t)$ for the USB signal.
      v. Repeat (iii) and (iv) to obtain the LSB signal $\phi_{LSB}(t)$.
      vi. Determine the time domain expressions $\phi_{LSB}(t)$ and $\phi_{USB}(t)$ if the carrier frequency $\omega_c = 1000$.

10. (a) Sketch $\phi_{FM}(t)$ and $\phi_{PM}(t)$ for the modulating signal $m(t)$ shown in Fig. 7, given $\omega_c = 10^8$, $k_f = 10^5$, and $k_p = 25$.
(b) Estimate the bandwidth for $\phi_{FM}(t)$ and $\phi_{PM}(t)$. Assume the bandwidth of $m(t)$ in Fig. 7 to be the third-harmonic frequency of $m(t)$.

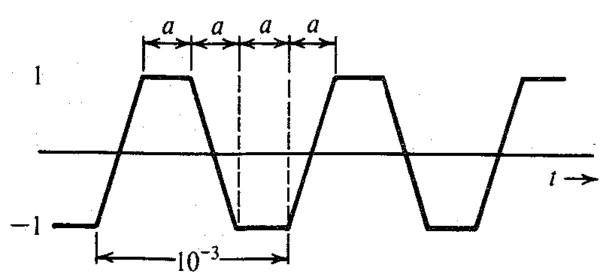

Figure 7:

11. A baseband signal $m(t)$ is the periodic sawtooth signal shown in Fig. 8.
   (a) Sketch $\phi_{FM}(t)$ and $\phi_{PM}(t)$ for this signal $m(t)$ if $\omega_c = 2\pi \times 10^6$, $k_f = 2000\pi$, and $k_p = \pi/2$.
   (b) Show that the PM signal is equivalent to a PM signal modulated by a rectangular periodic message. Explain why it is necessary to use $k_p < \pi$ in this case. [Note that the PM signal has a constant frequency but has phase discontinuities corresponding to the discontinuities of $m(t)$.]

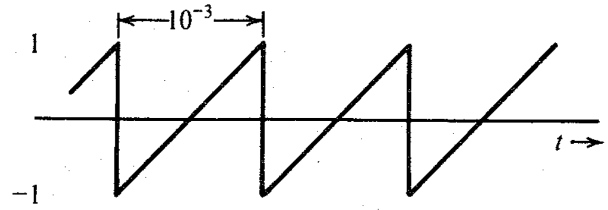

Figure 8:

(c) Estimate the bandwidth for $\phi_{FM}(t)$ and $\phi_{PM}(t)$. Assume the bandwidth of $m(t)$ in Fig. 8 to be the fifth-harmonic frequency of $m(t)$.

12. An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^6$ is described by the below equations. Considering one at a time, find,

$$\phi_{EM}(t) = 10\cos(\omega_c t + 0.1 \sin 2000\pi t).$$
$$\phi_{EM}(t) = 5\cos(\omega_c t + 20 \sin 1000\pi t + 10 \sin 2000\pi t).$$

(a) The power of the modulated signal.

(b) The frequency deviation $\triangle f$.

(c) The phase deviation $\triangle\phi$.

(d) The bandwidth of $\phi_{EM}(t)$.

13. A periodic square wave $m(t)$ (Fig. 9(a)) frequency-modulates a carrier of frequency $f_c = 10$ kHz with $\triangle_f = 1$ kHz. The carrier amplitude is A. The resulting FM signal is demodulated, as shown in Fig. 9(b). Sketch the waveforms at points b, c, d, and e. (Hint: Refer Sec. 5.4 (Fig. 5.12) of [B. P. Lathi, 4/e])

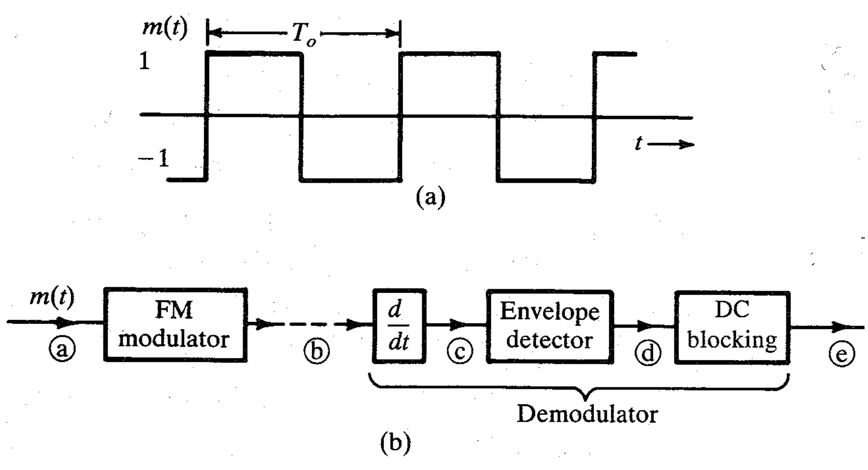

Figure 9: