Assignment 3
Communication Theory EE304

Submit Qts.: 2, 3, 6, 8, 10 and 11.

1. Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals:
   (a) $\text{sinc}(100\pi t)$;
   (b) $\text{sinc}^2(100\pi t)$;
   (c) $\text{sinc}(100\pi t)$+$\text{sinc}(50\pi t)$;
   (d) $\text{sinc}(100\pi t) + 3\text{sinc}^2(60\pi t)$;
   (e) $\text{sinc}(50\pi t)\text{sinc}(100\pi t)$;

2. A signal $g(t)$ =$\text{sinc}^2(5\pi t)$ is sampled (using uniformly spaced impulses) at a rate of (i) 5 Hz; (ii) 10 Hz; (iii) 20 Hz. For each of the three cases:
   (a) Sketch the sampled signal.
   (b) Sketch the spectrum of the sampled signal.
   (c) Explain whether you can recover the signal $g(t)$ from the sampled signal.
   (d) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the spectrum of the output signal.

3. Signals $g_l(t) = 10^4\text{rect}(10^4 t)$ and $g_2(t) = \delta(t)$ are applied at the inputs of ideal low-pass filters $H_1(f) = \text{rect}(\omega/40,000\pi)$ and $H_2(\omega) = \text{rect}(\omega/20,000\pi)$ (Fig. 1). The outputs $y_1(t)$ and $y_2(t)$ of these filters are multiplied to obtain the signal $y(t) = y_1(t)y_2(t)$. Find the Nyquist rate of $y_1(t), y_2(t)$, and $y(t)$. Use the convolution property and the width property of convolution to determine the bandwidth of $y_1(t)y_2(t)$.

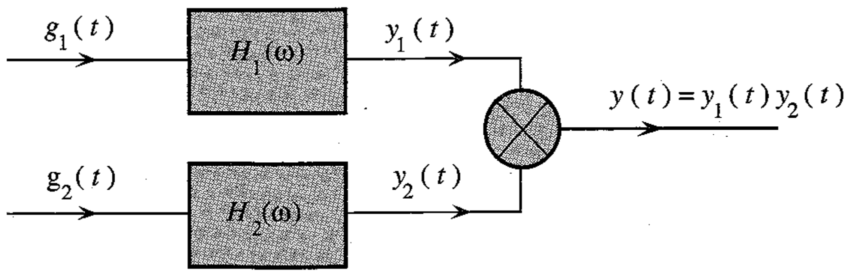

Figure 1:

4. Prove that a signal cannot be simultaneously time-limited and band-limited. Hint: Show that the contrary assumption leads to contradiction. Assume a signal simultaneously time-limited and band-limited so that $G(\omega) = 0$ for $|\omega| > 2\pi B$. In this case, $G(\omega) = G(\omega)\text{rect}(\omega/4B')$ for $B' > B$. This means that $g(t)$ is equal to $g(t) \star 2B' \text{ sinc}(2\pi B' t)$. Show that the latter cannot be time-limited.

5. The American Standard Code for Information Interchange (ASCII) has 128 characters, which are binary-coded. If a certain computer generates $100,000$ characters per second, determine the following:

   (a) The number of bits (binary digits) required per character.

   (b) The number of bits per second required to transmit the computer output, and the minimum bandwidth required to transmit this signal.

   (c) For single error detection capability, an additional bit (parity bit) is added to the code of each character. Modify your answers in parts (a) and (b) in view of this information.

6. A compact disc (CD) records audio signals digitally by using PCM. Assume that the audio signal bandwidth equals 15 kHz.

   (a) If the Nyquist samples are uniformly quantized into $L = 65,536$ levels and then binary-coded,determine the number of binary digits required to encode a sample.

   (b) If the audio signal has average power of 0.1 watt and peak voltage of 1 volt. Find the resulting signal-to-quantization-noise ratio (SQNR) of the uniform quantizer output in part (a).

   (c) Determine the number of binary digits per second (bit/s) required to encode the audio signal.

   (d) For practical reasons discussed in the text, signals are sampled at a rate well above the Nyquist rate. Practical CDs use $44,100$ samples per second. If $L = 65,536$, determine the number of bits per second required to encode the signal, and the minimum bandwidth required to transmit the encoded signal.

7. A television signal (video and audio) has a bandwidth of 4.5 MHz. This signal is sampled, quantized, and binary coded to obtain a PCM signal.

   (a) Determine the sampling rate if the signal is to be sampled at a rate 20% above the Nyquist rate.

   (b) If the samples are quantized into 1024 levels, determine the number of binary pulses required to encode each sample.

   (c) Determine the binary pulse rate (bits per second) of the binary-coded signal, and the minimum bandwidth required to transmit this signal.

8. A signal band-limited to 1 MHz is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 levels by using a μ-law quantizer with $\mu = 255$.

   (a) Determine the signal-to-quantization-noise ratio.

(b) The SQNR (the received signal quality) found in part (a) was unsatisfactory. It must be increased at least by 10 dB . Would you be able to obtain the desired SQNR without increasing the transmission bandwidth if it was found that a sampling rate 20% above the Nyquist rate is adequate? If so, explain how. What is the maximum SQNR that can be realized in this way?

9. The output SQNR of a 10-bit PCM ($n = 10$) was found to be insufficient at 30 dB . To achieve the desired SNR of 42 dB, it was decided to increase the number of quantization levels L. Find the fractional increase in the transmission bandwidth required for this increase in L.

10. Consider a full-width rectangular pulse shape $p(t) = \text{rect}(t/T_b)$.

    (a) Find PSDs for the polar, on-off, and bipolar signaling.

    (b) Sketch roughly the PSDs and find their bandwidths. For each case, compare the bandwidth to the case where p(t) is a half-width rectangular pulse.

11. (a) A random binary data sequence $100110\cdots$ is transmitted by using a Manchester (splitphase) line code with the pulse $p(t)$ shown in Fig. 2. Sketch the waveform $y(t)$.

    (b) Derive $S_y(\omega)$, the PSD of a Manchester (split-phase) signal in part (a) assuming 1 and 0 equally likely. Roughly sketch this PSD and find its bandwidth.

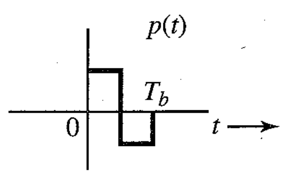

Figure 2:

12. Consider the two antipodal signal waveforms shown in Fig. 3. Show that these signal have exactly the same geometric representation as the two rectangular pulses in Fig. 4.

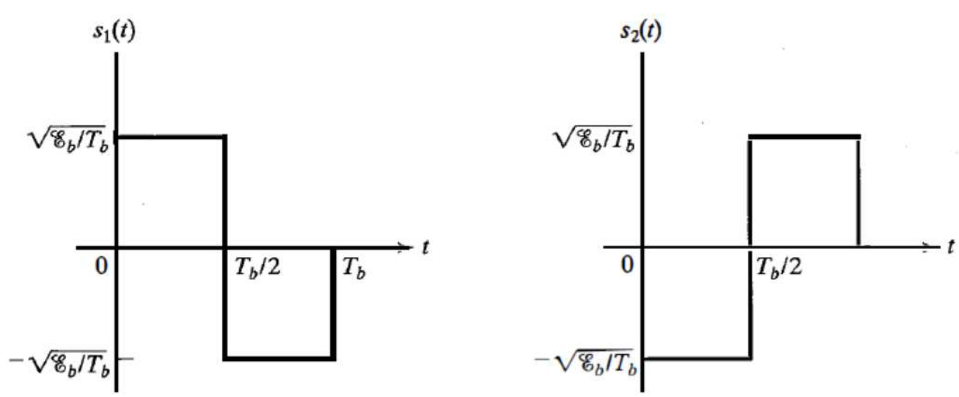

Figure 3:

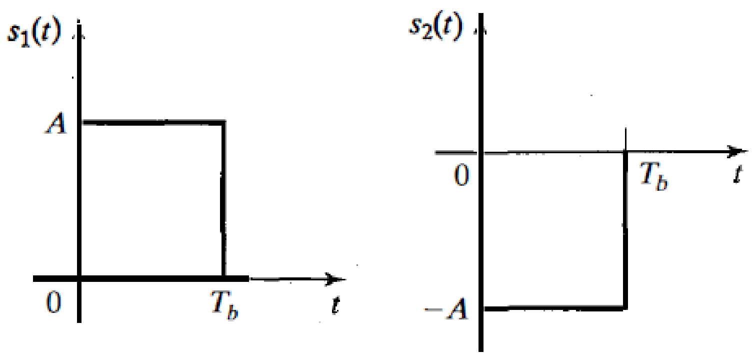

Figure 4: Binary PAM Signal