Subspace Based Multi-User Spectrum Sensing in Frequency Selective Cognitive Radio Systems

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***Abstract*—This paper proposes an optimal subspace based detector for multiple primary user spectrum sensing in frequency selective Rayleigh fading wireless channels. The proposed sensing scheme employs the optimal zero-forcing algorithm to null the effect of inter block interference (IBI) occurring due to the multipath nature of the wireless channel and also the multi user interference (MUI) arising due to the presence of other interfering primary users. This is followed by deriving the optimal Neyman-Pearson (NP) criterion based subspace detector to efficiently sense the presence/absence of the desired user. To characterize the efficacy of the proposed detection framework, expressions for the probabilities of false alarm and detection are also derived. Finally, performance comparison with existing detectors is presented.**

***Index Terms*—Cognitive radio, Spectrum Sensing, Zero-Forcing, Neyman-Pearson (NP)**

I. Introduction

Recent report [1] of the FCC reveals that a large portion of the licensed spectrum allotted to cellular users is underutilized due to the fixed spectrum assignment policy typically employed. Therefore, there is an urgent need for dynamic spectrum access which has paved the way for a new paradigm in modern wireless communications, termed cognitive radio [2]. It enables the unlicensed/ secondary user to utilize the spectrum space of licensed/ primary users without interfering with their transmission. Hence, spectrum sensing [3] to reliably identify the presence/ absence of primary user transmission, is a major challenge in the implementation of cognitive radio systems.

Existing literature on spectrum sensing techniques can be broadly classified into classical likelihood ratio test (LRT) based detection, matched filter detection, cyclostationary feature detection and energy detection. LRT [4], [5] is based on the optimal Neyman-Pearson (NP) criterion that computes the ratio of the likelihoods of the observations under both the hypotheses by utilizing the prior information of the signal, channel and noise statistics. Matched filtering [4] is also an optimal scheme for signal detection which requires a very small convergence time to achieve the desired detection performance. However, it is impractical since it requires perfect channel state information of the primary user. Further, in multi-user scenarios its detection performance degrades as it does not take into account multi-user interference (MUI). Cyclostationary feature detectors [6] are based on computing the cyclic autocorrelation of the received signal by exploiting the inherent cyclic features of the transmitted primary user signal depending on the modulation type, carrier frequency etc.. These detectors are found to be well suited for scenarios with low signal to noise power ratios, however at the cost of high computational complexity.

The energy detector [7]–[9] on the other hand is a widely used spectrum sensing algorithm due to its simplistic implementation, which only requires computation of the received signal energy in the time/ frequency domain. Therefore, energy detection based spectrum sensing procedures are well suited for scenarios without prior signal information. However for practical cellular scenarios in which digitally modulated narrowband signals are employed, the inherent knowledge of the signal characteristics needs to be exploited for better detection accuracy. In [10] and [11], the authors have proposed a spectrum sensing framework based on time and frequency domain energy detectors respectively. Although they have considered a frequency selective fading channel, the work therein does not consider inter-block interference (IBI). Further, some of the works such as [12], [13] have considered an OFDM based framework to mitigate the problem of IBI. However none of them have addressed the problem of IBI together with MUI. Further OFDM suffers from the problem of high peak to average power ratio and inter carrier interference due to frequency and timing offsets.

To address the above shortcomings, this paper presents a novel subspace spectrum sensing framework for the multiple primary user scenario which employs coherent combining based low complexity processing. Firstly, an optimal zero-forcing algorithm is proposed to null the effect of MUI caused due to other active primary user transmissions along with the IBI arising due to the frequency selective nature of practical wireless channels. This is followed by deriving the optimal NP criterion based subspace detector for primary user sensing. Results are also presented for the associated detection performance in terms of the probability of detection (P_D) and false alarm (P_{FA}).

II. System Model

Consider a spectrum sensing framework with M active primary users transmitting simultaneously at different carrier frequencies $(\acute{f_m})$, $1 \leq m \leq M$. The continuous time transmit signal $s_m(t)$ corresponding to the m^{th} primary user is,

$$s_m(t) = \sum_{\tilde{k}=-\infty}^{\infty} \acute{I}_{\tilde{k},m} g_m\left(t - \tilde{k}T_b\right) e^{j2\pi \acute{f_m} t}, \qquad (1)$$

The first two authors have contributed equally to this work.

where $\acute{I}_{\tilde{k},m}$ denotes the $\tilde{k}^{th}$ information symbol drawn from a constellation with average power $\acute{P}_m = \mathrm{E}\{|\acute{I}_{\tilde{k},m}|^2\}$. T_b is the symbol duration and $g_m(t)$ represents the impulse response of the transmit pulse shaping filter with pulse spread $T' \geq T_b$. The sampled signal $s_m(lT_s)$ is represented as,

$$s_m(lT_s) = \sum_{\tilde{k}=-\infty}^{\infty} \acute{I}_{\tilde{k},m} g_m\left(lT_s - \tilde{k}T_b\right) e^{j2\pi \frac{f_m}{L} l}, \quad (2)$$

where L denotes the number of samples in a symbol duration $[kT_b, (k+1)T_b)$, $f_m = \acute{f}_m T_s L$ and sampling duration $T_s = \frac{1}{f_s} = \frac{T'}{L'} = \frac{T_b}{L}$ with L' denoting the number of samples per symbol after pulse shaping. The l^{th} sample of the k^{th} symbol corresponding to the time instant $kT_b + lT_s$, denoted by $s_{k,m}(l) = s_m(kT_b + lT_s)$, is given as,

$$s_{k,m}(l) = \sum_{\tilde{k}=-\infty}^{\infty} \acute{I}_{\tilde{k},m} g_m\left(kT_b + lT_s - \tilde{k}T_b\right) e^{j2\pi \frac{f_m}{L}(kL+l)} \quad (3)$$

Since $g_m(iT_s) \neq 0, \ \forall \ 0 \leq i \leq (L'-1)$, implies $g_m\left(kT_b + lT_s - \tilde{k}T_b\right) \neq 0, \ \forall \ \left\lceil \frac{kL+l-L'+1}{L} \right\rceil \leq \tilde{k} \leq \left\lfloor \frac{kL+l}{L} \right\rfloor$, where $\lceil . \rceil$ and $\lfloor . \rfloor$ denote the ceiling and floor functions respectively. Hence the expression for $s_{k,m}(l)$ above simplifies to,

$$s_{k,m}(l) = \frac{1}{\sqrt{L}} \sum_{\tilde{k}=\left\lceil \frac{kL+l-L'+1}{L} \right\rceil}^{\left\lfloor \frac{kL+l}{L} \right\rfloor} I_{\tilde{k},m} g_m\left(lT_s + (k-\tilde{k})T_b\right) \times e^{j2\pi \frac{f_m}{L} l}, \quad (4)$$

where $I_{\tilde{k},m} = \sqrt{L}\acute{I}_{\tilde{k},m}$, with average power $P_m = \acute{P}_m L$.

Consider a finite impulse response (FIR) block fading frequency selective Rayleigh fading wireless channel modeled as $h_{k,m}(i) \sim \mathcal{CN}(0, \rho), \ 0 \leq i \leq \nu - 1$ with uniform power delay profile, where ν denotes the number of channel taps and $\rho = \frac{1}{\nu}$ denotes the average gain of the channel taps. Assume that the channel is independent of the information symbols, from which it follows that $\mathrm{E}\left\{h_{k,m}(i) I_{\tilde{k},m}\right\} = \mathrm{E}\{h_{k,m}(i)\} \mathrm{E}\left\{I_{\tilde{k},m}\right\}$. The aggregate signal $r_k(l)$ received at the secondary user is given as,

$$r_k(l) = \sum_{m=1}^{M} \sum_{i=0}^{\nu-1} s_{k,m}(l - l_0 - i) h_{k,m}(i) + \delta_k(l), \quad (5)$$

where $\delta_k(l) \sim \mathcal{CN}\left(0, \sigma^2\right)$ denote the AWGN samples and l_0 is the delay experienced by the transmit signal. Using equation (4), the received signal $r_k(l)$ above can be modified as,

$$r_k(l) = \sum_{m=1}^{M} \sum_{i=0}^{\nu-1} \sum_{\tilde{k}=k+\left\lceil \frac{l-l_0-i-L'+1}{L} \right\rceil}^{k+\left\lfloor \frac{l-l_0-i}{L} \right\rfloor} I_{\tilde{k},m} g_m\left((l-l_0-i)T_s + \left(k-\tilde{k}\right)T_b\right) \times e^{j2\pi \frac{f_m}{L}(l-l_0-i)} h_{k,m}(i) + \delta_k(l). \quad (6)$$

Therefore, $\mathbf{r}_k \in \mathbb{C}^{L\times 1} = \left[r_k(0), \cdots, r_k(L-1)\right]^T$ corresponding to the time interval $[kT_b, (k+1)T_b)$ is,

$$\mathbf{r}_k = \boldsymbol{\delta}_k + \sum_{m=1}^{M} \mathbf{A}(f_m, l_0) \mathbf{h}_{k,m} I_{k,m} + \sum_{m=1}^{M} \sum_{\substack{j=1 \\ j\neq 0}}^{J} \mathbf{B}_j(f_m, l_0) \mathbf{h}_{k,m} I_{k-j,m}, \quad (9)$$

where $\boldsymbol{\delta}_k = \left[\delta_k(0), \cdots, \delta_k(L-1)\right]^T$ represents the AWGN vector, $J = \left\lfloor \frac{L'+\nu-2}{L} \right\rfloor$ is the total number of interfering symbols and $\mathbf{h}_{k,m} = \left[h_{k,m}(0), \cdots, h_{k,m}(\nu-1)\right]^T \in \mathbb{C}^{\nu\times 1}$ denotes the channel vector corresponding to the k^{th} symbol block of the m^{th} primary user. The parametrized signal and interference subspace matrices $\mathbf{A}(f_m, l_0) \in \mathbb{C}^{L\times 1}$, $\mathbf{B}_j(f_m, l_0) \in \mathbb{C}^{L\times 1}$ respectively, are given as,

$$\mathbf{A}(f_m, l_0) = \left[\mathbf{v}_0(f_m, l_0), \mathbf{v}_0^{(1)}(f_m, l_0), \ldots, \mathbf{v}_0^{(\nu-1)}(f_m, l_0)\right],$$
$$\mathbf{B}_j(f_m, l_0) = \left[\mathbf{v}_j(f_m, l_0), \mathbf{v}_j^{(1)}(f_m, l_0), \ldots, \mathbf{v}_j^{(\nu-1)}(f_m, l_0)\right], \quad (10)$$

where $\mathbf{v}_j(f_m, l_0) \in \mathbb{C}^{L\times 1} = \mathbf{G}_j(l_0) \mathbf{w}(f_m, l_0), 0 \leq j \leq J$ with $\left[\mathbf{G}_j(l_0)\right]_i = g_m((i-l_0)T_s + jT_b)$ denoting the i^{th} entry of the diagonal pulse shaping filter matrix $\mathbf{G}_j(l_0) \in \mathbb{C}^{L\times L}$ and the vector $\mathbf{w}(f_m, l_0)$ given as,

$$\mathbf{w}(f_m, l_0) = \frac{\omega(l_0)}{\sqrt{L}} \left[e^{j2\pi \frac{f_m}{L}(0)}, e^{j2\pi \frac{f_m}{L}(1)}, \ldots, e^{j2\pi \frac{f_m}{L}(L-1)}\right]^{\mathrm{T}}, \quad (11)$$

where $\omega(l_0) = e^{-j2\pi \frac{f_m}{L} l_0}$. The vectors $\mathbf{v}_j^{(i)}(f_m, l_0), \ \forall j$ are defined in (7) along with $a(j,i) = g_m((jL-i)T_s)\, e^{j2\pi \frac{f_m}{L}(jL-i)}$ and $\mathbf{e}_j = [0 \cdots 1 \cdots 0]^T \in \mathbb{C}^{L\times 1}$ denotes the vector with 1 at the j^{th} index.

III. Proposed Zero-Forcing Algorithm

The k^{th} received symbol block $\mathbf{r}_k$ defined in (9) can be resolved into its components along the signal, interference and noise subspaces as shown below,

$$\mathbf{r}_k = \mathbf{A}(f_m, l_0) \mathbf{h}_{k,m} I_{k,m} + \boldsymbol{\delta}_k + \sum_{\substack{\tilde{m}=1 \\ \tilde{m}\neq m}}^{M} \mathbf{A}(f_{\tilde{m}}, l_0) \mathbf{h}_{k,\tilde{m}} I_{k,\tilde{m}} + \sum_{\tilde{m}=1}^{M} \sum_{\substack{j=1 \\ j\neq 0}}^{J} \mathbf{B}_j(f_{\tilde{m}}, l_0) \mathbf{h}_{k,\tilde{m}} I_{k-j,\tilde{m}}, \quad (12)$$

where $\mathbf{A}(f_m, l_0) \in \mathbb{C}^{L\times\nu}$ represents the desired user signal basis matrix, $\mathbf{A}(f_{\tilde{m}}, l_0) \in \mathbb{C}^{L\times\nu}, \ \forall \ \tilde{m} \neq m$ denotes the subspace of the interfering users corresponding to the k^{th} symbol block and $\mathbf{B}_j(f_{\tilde{m}}, l_0) \in \mathbb{C}^{L\times\nu}, \ \forall \ 1 \leq \tilde{m} \leq M$ represents the interference subspace corresponding to the $(k-j)^{th}$ symbol. Let $\mathbf{b}_k(f_m, l_0)$ denote the unit norm beamforming vector corresponding to the k^{th} symbol block of the m^{th} user. The output $z_{k,m}$ after beamforming is given as,

$$z_{k,m} = \mathbf{b}_k^H(f_m, l_0) \mathbf{r}_k, \quad (13)$$

where $\mathbf{r}_k$ is defined in (12). Since $\mathbf{A}(f_{\tilde{m}}, l_0), \ \tilde{m} \neq m$ and $\mathbf{B}_j(f_{\tilde{m}}, l_0)$ together constitute the interference subspace of the k^{th} symbol block corresponding to the m^{th} primary user, the combined interference subspace matrix $\mathbf{C}(f_m, l_0) \in \mathbb{C}^{L\times(JM+M-1)\nu} = [\mathcal{A}(f_m, l_0) \ \ \mathcal{B}(f_m, l_0)]$,

$$\mathbf{v}_0^{(i)}(f_m,l_0) = \left[\mathbf{0}_{i\times 1}^{\mathrm{T}}, \mathbf{e}_1^{\mathrm{T}}\mathbf{v}_0(f_m,l_0), \mathbf{e}_2^{\mathrm{T}}\mathbf{v}_0(f_m,l_0), \cdots, \mathbf{e}_{L-i}^{\mathrm{T}}\mathbf{v}_0(f_m,l_0)\right]^{\mathrm{T}},$$
$$\mathbf{v}_j^{(i)}(f_m,l_0) = \left[a(j,i), a(j,i-1)\cdots, a(j,1), \mathbf{e}_1^{\mathrm{T}}\mathbf{v}_j(f_m,l_0), \cdots, \mathbf{e}_{L-i}^{\mathrm{T}}\mathbf{v}_j(f_m,l_0)\right]^{\mathrm{T}}. \quad (7)$$
$$\mathcal{A}(f_m,l_0) = [\mathbf{A}(f_1,l_0), \cdots, \mathbf{A}(f_{m-1},l_0), \mathbf{A}(f_{m+1},l_0), \cdots, \mathbf{A}(f_M,l_0)],$$
$$\mathcal{B}(f_m,l_0) = [\mathbf{B}_1(f_1,l_0), \cdots, \mathbf{B}_J(f_1,l_0), \cdots\cdots, \mathbf{B}_1(f_M,l_0), \cdots, \mathbf{B}_J(f_M,l_0)]. \quad (8)$$

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where the matrices $\mathcal{A}(f_m,l_0)$ and $\mathcal{B}(f_m,l_0)$ are defined as in (8). The m^{th} user signal power $\varsigma_m = \mathrm{E}\left\{|\mathbf{b}_k^H(f_m,l_0)\mathbf{A}(f_m,l_0)\mathbf{h}_{k,m}I_{k,m}|^2\right\}$ corresponding to the k^{th} symbol block is given as,

$$\begin{aligned}\varsigma_m =& \left(\mathbf{b}_k^H(f_m,l_0)\mathbf{A}(f_m,l_0)\right)\mathrm{E}\left\{\mathbf{h}_{k,m}I_{k,m}I_{k,m}^*\mathbf{h}_{k,m}^H\right\}\\ &\times\left(\mathbf{b}_k^H(f_m,l_0)\mathbf{A}(f_m,l_0)\right)^H\\ =& P_m\rho\left\|\mathbf{b}_k^H(f_m,l_0)\mathbf{A}(f_m,l_0)\right\|_2^2.\end{aligned} \quad (14)$$

Similarly the noise power $\mathrm{E}\left\{|\mathbf{b}_k^H(f_m,l_0)\boldsymbol{\delta}_k|^2\right\} = \sigma^2$ which follows from the fact that $\|\mathbf{b}_k(f_m,l_0)\|_2^2 = 1$. Since the parameters P_m, ρ and σ^2 are constant, maximizing the SNR $= \frac{1}{\sigma^2}P_m\rho\left\|\mathbf{b}_k^H(f_m,l_0)\mathbf{A}(f_m,l_0)\right\|_2^2$ is equivalent to maximizing $\left\|\mathbf{b}_k^H(f_m,l_0)\mathbf{A}(f_m,l_0)\right\|_2^2$. Also, in order to null the interference from the other $M-1$ users the desired constraint is $\mathbf{b}_k^H(f_m,l_0)\mathbf{C}(f_m,l_0) = \mathbf{0}_{(JM+M-1)\nu\times 1}^{\mathrm{T}}$. Hence the optimization problem to determine $\mathbf{b}_k(f_m,l_0)$ can be formulated as follows,

$$\begin{aligned}\max_{\mathbf{b}_k(f_m,l_0)} \quad & \left\|\mathbf{b}_k^H(f_m,l_0)\mathbf{A}(f_m,l_0)\right\|_2^2,\\ \text{s. t.} \quad & \mathbf{b}_k^H(f_m,l_0)\mathbf{C}(f_m,l_0) = \mathbf{0}_{(JM+M-1)\nu\times 1}^{\mathrm{T}},\\ & \|\mathbf{b}_k(f_m,l_0)\|_2^2 = 1.\end{aligned} \quad (15)$$

Define $\mathbf{b}_k(f_m,l_0)=\mathbf{C}^{\perp}(f_m,l_0)\mathbf{u}_k(f_m,l_0)$, where $\mathbf{C}^{\perp}(f_m,l_0)$ denotes a basis for the null space of $\mathbf{C}(f_m,l_0)$ and $\mathbf{u}_k(f_m,l_0)$ is any vector satisfying $\|\mathbf{u}_k(f_m,l_0)\|_2^2 = 1$. Thus, the equivalent optimization problem is,

$$\begin{aligned}\max_{\mathbf{u}_k(f_m,l_0)} \quad & \mathbf{u}_k^H(f_m,l_0)\mathbf{S}(f_m,l_0)\mathbf{S}^H(f_m,l_0)\mathbf{u}_k(f_m,l_0),\\ \text{s. t.} \quad & \mathbf{u}_k^H(f_m,l_0)\mathbf{u}_k(f_m,l_0) = 1.\end{aligned}$$

where $\mathbf{S}(f_m,l_0) = (\mathbf{C}^{\perp}(f_m,l_0))^H\mathbf{A}(f_m,l_0)$. The solution $\mathbf{u}_k^o(f_m,l_0)$ to the above optimization problem is the eigenvector corresponding to the maximum eigenvalue of the matrix $\mathbf{S}(f_m,l_0)\mathbf{S}^H(f_m,l_0)$ which thus yields the the optimal unit-norm zero forcing beamforming vector $\mathbf{b}_k(f_m,l_0)$ as,

$$\mathbf{b}_k(f_m,l_0) = \mathbf{C}^{\perp}(f_m,l_0)\mathbf{u}_k^o(f_m,l_0). \quad (16)$$

It can be seen that the matrix $\mathbf{S}(f_m,l_0)\mathbf{S}^H(f_m,l_0)$ is independent of the time index k which implies $\mathbf{u}_k^o(f_m,l_0)$, $\mathbf{b}_k(f_m,l_0)$ are also independent of time index k. Therefore we have $\mathbf{b}_k(f_m,l_0) = \mathbf{b}(f_m,l_0), \forall k$. Thus, the decoupled binary hypothesis testing problem after beamforming with $\mathbf{b}(f_m,l_0)$ to detect the presence/ absence of the m^{th} primary user corresponding to the k^{th} symbol block is given as,

$$\begin{aligned}&\mathcal{H}_0 : z_{k,m} = \delta_{k,m}, \quad 1\le m\le M, 0\le k\le K-1\\ &\mathcal{H}_1 : z_{k,m} = \mathbf{b}^H(f_m,l_0)\mathbf{A}(f_m,l_0)\mathbf{h}_{k,m}I_{k,m} + \delta_{k,m}.\end{aligned} \quad (17)$$

IV. Subspace Based Spectrum Sensing

Observe that under hypothesis $\mathcal{H}_0$, the received signal $z_{k,m} = \delta_{k,m}$ in (17) is distributed as $\mathcal{CN}\left(0,\sigma^2\right)$. Similarly, under $\mathcal{H}_1$ the distribution of observation quantity $z_{k,m} = \mathbf{b}^H(f_m,l_0)\mathbf{A}(f_m,l_0)\mathbf{h}_{k,m}I_{k,m} + \delta_{k,m}$ in (17) is again Gaussian with mean $\mathrm{E}\{z_{k,m}\}$ and variance $\mathrm{E}\{|z_{k,m}|^2\}$ derived as,

$$\begin{aligned}\mathrm{E}\{z_{k,m}\} &= \mathbf{b}^H(f_m,l_0)\mathbf{A}(f_m,l_0)\mathrm{E}\{\mathbf{h}_{k,m}\}\mathrm{E}\{I_{k,m}\} + \mathrm{E}\{\delta_{k,m}\}\\ &= 0,\\ \mathrm{E}\{|z_{k,m}|^2\} &= \mathrm{E}\{|\mathbf{b}^H(f_m,l_0)\mathbf{A}(f_m,l_0)\mathbf{h}_{k,m}I_{k,m}|^2\}\\ &\quad + \mathrm{E}\{|\delta_{k,m}|^2\} = \varsigma_m + \sigma^2,\end{aligned} \quad (18)$$

where the quantity ς_m is derived in equation (14). Therefore, the optimal NP criterion based LRT for the observation block $\mathbf{z}_m = [z_{0,m}, \cdots, z_{K-1,m}]^{\mathrm{T}}$, decides $\mathcal{H}_1$ if,

$$\begin{aligned}\mathcal{L}(\mathbf{z}_m) &= \frac{\prod_{k=0}^{K-1}\wp(z_{k,m};\mathcal{H}_1)}{\prod_{k=0}^{K-1}\wp(z_{k,m};\mathcal{H}_0)} > \gamma,\\ &= \frac{\prod_{k=0}^{K-1}\frac{1}{\pi(\sigma^2+\varsigma_m)}e^{-\frac{|z_{k,m}|^2}{(\sigma^2+\varsigma_m)}}}{\prod_{k=0}^{K-1}\frac{1}{\pi\sigma^2}e^{-\frac{|z_{k,m}|^2}{\sigma^2}}} > \gamma,\end{aligned} \quad (19)$$

where $\wp$ denotes the likelihood function. Taking the logarithm on both sides and simplifying, the LRT above simplifies to,

$$\tilde{\mathcal{L}}(\mathbf{z}_m) = \sum_{k=0}^{K-1}|z_{k,m}|^2 > \tilde{\gamma}, \quad (20)$$

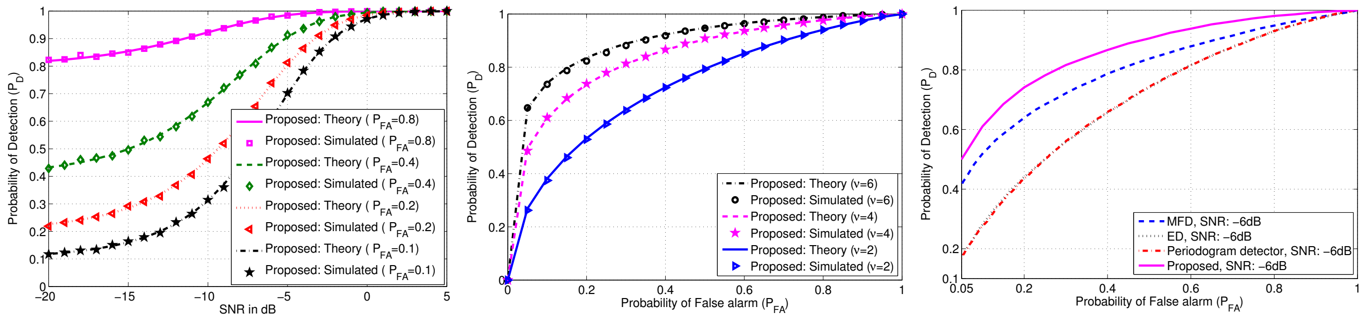
where $\tilde{\gamma} = \frac{\sigma^2(\sigma^2+\varsigma_m)}{\varsigma_m}\left(\ln\gamma - K\ln\left(\frac{\sigma^2}{\sigma^2+\varsigma_m}\right)\right)$ is the modified detection threshold and $z_{k,m} = \mathbf{b}^H(f_m,l_0)\mathbf{r}_k$. The optimal LRT test statistic $T_m(\mathbf{R})$ is now obtained as,

$$T_m(\mathbf{R}) = \sum_{k=0}^{K-1}\left|\mathbf{b}^H(f_m,l_0)\mathbf{r}_k\right|^2 = \sum_{k=0}^{K-1}\mathbf{r}_k^H\mathbf{P}(f_m,l_0)\mathbf{r}_k, \quad (21)$$

where $\mathbf{R} = [\mathbf{r}_0,\ldots,\mathbf{r}_{K-1}] \in \mathbb{C}^{L\times K}$ and $\mathbf{P}(f_m,l_0) = \mathbf{b}(f_m,l_0)\mathbf{b}^H(f_m,l_0) \in \mathbb{C}^{L\times L}$ is the rank-one projection matrix with respect to $\mathbf{b}(f_m,l_0)$ along the m^{th} user signal subspace. Therefore, the above test statistic $T_m(\mathbf{R})$ is a subspace based detector, the distribution of which is derived as follows. Since the observation quantity $z_{k,m}$ in (13) is distributed as $\mathcal{CN}\left(0,\sigma^2\right)$, $\mathcal{CN}\left(0,\varsigma_m+\sigma^2\right)$ under hypotheses $\mathcal{H}_0$, $\mathcal{H}_1$ respectively, the distributions of the test statistic $T_m(\mathbf{R})$ can be obtained as shown below,

$$T_m(\mathbf{R}) \sim \begin{cases}\chi_{2K}^2 \text{ with variance } \frac{1}{2}\sigma^2, \ \mathcal{H}_0,\\ \chi_{2K}^2 \text{ with variance } \frac{1}{2}\left(\varsigma_m+\sigma^2\right), \ \mathcal{H}_1,\end{cases} \quad (22)$$

where $\mathcal{X}_{2K}^2$ denotes a central chi-squared random variable with $2K$ degrees of freedom.



(a) (b) (c)

Fig. 1. (a) P_D versus SNR performance of the proposed subspace based detector for different values of P_{FA} at $\nu = 4$, (b) Receiver operating characteristics with varying number of channel taps (ν) at SNR $=-6$dB, (c) Performance comparison with the existing detectors.

Also, the associated performance in terms of the probability metrics P_{FA}, P_D is derived as,

$$\begin{aligned} P_{FA} &= \Pr\left(T_m(\mathbf{R}) > \tilde{\gamma}; \mathcal{H}_0\right) = Q_{\chi^2_{2K}}\left(\frac{2\tilde{\gamma}}{\sigma^2}\right), \\ P_D &= \Pr\left(T_m(\mathbf{R}) > \tilde{\gamma}; \mathcal{H}_1\right) = Q_{\chi^2_{2K}}\left(\frac{2\tilde{\gamma}}{\varsigma_m + \sigma^2}\right). \end{aligned} \tag{23}$$

Combining the above expressions yields the corresponding receiver operating characteristic as,

$$P_D = Q_{\chi^2_{2K}}\left(\frac{\sigma^2}{\sigma^2 + \varsigma_m} Q^{-1}_{\chi^2_{2K}}\left(P_{FA}\right)\right). \tag{24}$$

V. Numerical Results

This section illustrates the performance of the proposed subspace based detection scheme. A multiple primary user scenario with $M = 3$ primary users transmitting $K = 5$, QPSK modulated symbols at carrier frequencies $\acute{f}_1 = 180$, $\acute{f}_2 = 210$, and $\acute{f}_3 = 240$ MHz respectively is considered. The average transmit power of each user is $P_m = 1W$. The parameters T_b and T_s are set as $0.2\mu s$ and $0.002\mu s$ respectively such that $L = 100$. The standard Sinc filter with response $g_m(t) = \frac{1}{T'}\text{sinc}\left(\frac{t}{T'}\right)$ is used with normalized power $\frac{1}{T'}\int_{t=-\frac{T'}{2}}^{\frac{T'}{2}} |g_m(t)|^2 dt = 1$. The length of the pulse shaping filter T' is assumed to be $T' = T_b$ such that $L = L' = 100$.

Fig. 1a illustrates the P_D versus SNR performance of the subspace based test statistic in (21) for varying P_{FA} at $\nu = 4$. As expected the P_D increases with increasing P_{FA} at a given SNR. Also, the analytical expression derived in (23) is found to closely match with the simulated performance. Fig. 1b demonstrates the receiver operating characteristic of the proposed detector with varying number of channel taps (ν) at a fixed SNR of $-6dB$. Since the channel taps are independent and identically distributed (i.i.d.) Gaussian random variables, increasing ν will increase the diversity order of the wireless channel thereby improving the sensing accuracy. Finally, Fig. 1c, shows the improved detection performance of the proposed subspace based detector in comparison to the conventional time domain energy detector, periodogram spectral detector and matched filter detector. This is due to the fact that the proposed framework nulls the effect of both inter block as well as inter user interference using the optimal zero forcing algorithm derived in section III.

VI. Conclusion

This paper proposes an optimal subspace based detector for sensing the desired primary user spectrum in the presence of multiple interfering primary users over frequency selective fading wireless channels. The analytical expressions for the detection metrics P_D, P_{FA} are also derived. Simulation results demonstrate the improved detection performance in comparison to the existing detectors and validate the derived analytical expressions.

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