

# Robust Cooperative Spectrum Sensing for MIMO Cognitive Radio Networks under CSI Uncertainty

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**Abstract**—This paper considers the problem of cooperative spectrum sensing in multiuser multiple-input multiple-output (MIMO) cognitive radio networks considering the presence of uncertainty in the channel state information (CSI) of the secondary user channels available at the fusion center. Several schemes are proposed that employ cooperative decision rules based on local sensor decisions transmitted to the fusion center by the cooperating nodes over an orthogonal multiple access channel (MAC). First, fusion rules are derived under perfect CSI at the fusion center for both antipodal and non-antipodal signaling. Then, a robust detector, termed the uncertainty statistics-based likelihood ratio test (US-LRT), which optimally combines the decisions of different secondary users, is obtained for scenarios with CSI uncertainty. A generalized likelihood ratio test (GLRT) based robust detector is also derived for this scenario. Closed form expressions are obtained to characterize the probabilities of false alarm ( $P_{FA}$ ) and detection ( $P_D$ ) at the fusion center. Simulation results are presented to compare the performance of the proposed schemes with that of the conventional uncertainty agnostic detectors and also to corroborate the analytical expressions developed.

**Index Terms**—Cognitive radio networks, cooperative spectrum sensing, multiple-input multiple-output (MIMO), generalized likelihood ratio test (GLRT), likelihood ratio test (LRT).

## I. INTRODUCTION

### A. Motivation and Related Work

Recent years have witnessed an increasing demand for higher data rates to support a variety of advanced wireless applications leading to new challenges in wireless networks with limited radio resources. Interestingly, several reports [1]–[3], including one by the Federal Communications Commission (FCC), reveal that the apparent scarcity of spectrum arises from inefficient utilization of existing radio spectrum rather than its paucity. Towards this end, the newly emerging paradigm of cognitive radio (CR) technology [4], [5] increases the efficiency of spectrum utilization by enabling dynamic spectrum access (DSA) by a limited set of *unlicensed* or *secondary* users. Thus, it is essential for the cognitive radio user, i.e., the secondary user, to *sense* the spectrum to determine the availability of vacant bands, known also as spectrum holes, in order to opportunistically access resources temporarily unused by the licensed primary users [6]. Several works in the existing literature [4]–[11] have proposed various schemes for spectrum sensing in cognitive radio networks. Further, results in [8]–[11] demonstrate that cooperation between multiple-users,

also known as cooperative spectrum sensing (CSS), and/or multiple-antennas [12]–[14], significantly improves the sensing performance in comparison to single user single antenna detection schemes. For cooperative sensing scenarios [15], [16] with a large number of users, transmission of local decisions to the fusion center, rather than analog measurements, significantly reduces the bandwidth required for the reporting channels [8] while leading to only a minor deterioration in detection performance. As described in the original work in [16], analog sensor observations or soft information can be transmitted to the fusion center. However, this requires the transmission of local sensor information without delay together with a large communication bandwidth. Further, the associated signal processing schemes are based on classical signal processing theory and tend to be complex. In contrast, in distributed signal processing, some initial signal processing is carried out by the sensors is followed by the transmission of local decisions to a fusion center for further processing. Such a scheme is attractive for practical implementation due to its many advantages such as low cost, improved reliability, fault tolerance, scalability, energy efficiency and limited bandwidth requirements.

In this context, schemes have been presented in the literature for the fusion of local decisions received from the cooperating sensors in a wireless sensor network in [12], [14], [17], [18] and similarly for local decisions received from the secondary users in a cognitive radio network in [10], [11], [19]. In the work in [17], the authors employ the Chair-Varshney (CV) rule [16], maximum ratio combining (MRC) and, equal gain combining (EGC) techniques for coherent combining at the fusion center in additive white Gaussian noise (AWGN) wireless sensor networks. Works [16], [17] consider wireless scenarios by incorporating channel fading, while authors in [18] and [20] additionally develop fusion rules for scenarios where only statistics of the fading channel is known. Further, works such as [10], [11], [19] employ suboptimal energy detection at the sensors for cooperative spectrum sensing at the fusion center. The work in [21] considers spectrum sensing for cognitive radio scenarios with multiple antennas at the fusion center and single antennas at the individual sensors for both cooperative/ non-cooperative transmissions. The work in [22] proposes a scheme for coherent decision fusion for wireless sensor networks with single antenna sensors that forward their decisions based on a max-log approximation to the optimal fusion rule. Further, the MRC fusion rule is also obtained assuming that the sensors are reliable, i.e.,  $P_D = 1$ ,  $P_{FA} = 0$ . A semi-theoretical analysis is presented therein based on the Laplace transform of the pdf. Authors in [23] derived the maximum ratio combining (MRC) rule for a MIMO multiple access

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channel (MAC) for conditionally dependent and independent local decisions. Analytical results for probabilities of false alarm and detection are derived conditioned on the channel realization followed by the conditional moment generating function (MGF) of the MRC statistic with channel averaging. Moreover, nearly all the works surveyed above for coherent cooperative detection such as [10], [11], [16]–[19], [24], [25] consider single-input single-output (SISO), i.e., single antenna equipped nodes.

Coming now to the CSI aspect, the detectors for cooperative sensing in [12], [14], [15], [17], [24] require perfect CSI for the reporting links between the cooperating sensors/secondary users and the fusion center, i.e., they are inherently CSI dependent. However, acquisition of accurate CSI in such cooperative wireless scenarios is difficult and imposes unrealistic overheads. As the works in [13], [26], [27] have shown, the detection performance for such cooperative schemes critically depends on the accuracy of the obtained CSI. The work in [13] presents an amplify and forward scheme to relay the observations to the fusion center considering both the unknown and known CSI cases in a wireless sensor network. In [28], decision fusion (DF) is analyzed for a wireless sensor network (WSN) with multiple antennas when only statistical channel information is available at the fusion center and local decisions are transmitted over Rician channels. Sub-optimal fusion rules are obtained for this scenario and the model is also extended to a scenario with jammer interference. In [29], a study of channel-aware decision fusion is presented for a “virtual” multiple-input multiple-output (MIMO) system with a massive antenna array at the fusion center. The analysis therein considers linear fusion rules with imperfect CSI followed by the pertinent performance and complexity analyses. However, the detectors under unknown CSI require either a strong line-of-sight component or are based on complex iterative procedures with approximations that lead to suboptimality. Though the work in [26] considers channel imperfections, it employs the simplistic energy detector without deriving the optimal decision rules at the sensors for such a scenario. Our previous work in [27] considered spectrum sensing for single user MIMO systems with CSI uncertainty and presented robust detectors, which outperform other CSI uncertainty agnostic sensing schemes, together with the pertinent analysis. This work, which considers a multi-user cooperative MIMO spectrum sensing scenario and derives optimal fusion rules based on local sensor decisions, represents a paradigm shift from the previous work. Additionally, in contrast to the previous work, the current work considers scenarios with perfect as well as imperfect CSI for both antipodal and non-antipodal signaling formats.

Related works such as [11], [19] consider error free transmission of local decisions made by the individual sensors/secondary users. It once again goes without saying that such an assumption is unrealistic for practical sensing scenarios in which the local decisions are received with a finite error rate at the fusion center. To summarize, cooperative spectrum sensing under imperfect CSI of channels between the secondary users and the fusion center coupled with potentially erroneous local decisions is a significantly challenging task, while being very

relevant practically.

## B. Main Contributions and Paper Organisation

This work presents schemes for cooperative spectrum sensing in multi-user MIMO cognitive radio networks considering the availability of both perfect CSI and CSI with uncertainty. The multiple cooperating users are assumed to transmit only local decisions over either parallel access channels or orthogonal MAC, which are susceptible to errors with a finite probability, thus making the setup more practical. The multiple observation vector based cooperative sensing model for multiuser MIMO cognitive radio networks is described in Section II followed by the optimal likelihood ratio test (LRT) based sensing schemes with perfect CSI between the secondary transmitter and fusion center for both antipodal and non-antipodal signaling schemes in Section III. Next, the *uncertainty statistics* based likelihood ratio test (US-LRT) and robust generalized likelihood ratio test (R-GLRT) are determined for cooperative spectrum sensing scenarios under CSI uncertainty and are presented in Sections IV and V respectively. Closed form analytical expressions are also obtained for the probability of detection ( $P_D$ ) and the probability of false alarm ( $P_{FA}$ ) to characterize the performance of the proposed detectors. Simulation results demonstrate the improved performance of the proposed robust detectors in comparison to the conventional CSI uncertainty agnostic detectors, while also verifying the analytical results in Section VI. The paper concludes with Section VII.

A brief description of the notation follows. Boldface lowercase letters are employed to denote vectors and boldface uppercase letters to denote matrices. The operations  $\exp\{\cdot\}$ ,  $E\{\cdot\}$ ,  $|\cdot|$ ,  $\Re(\cdot)$ ,  $(\cdot)^*$ ,  $(\cdot)^H$ ,  $(\cdot)^T$  and  $\|\cdot\|$  denote the exponential function, expectation operator, determinant, real part, conjugate, conjugate transpose, transpose and norm respectively.  $p(\cdot)$  represents probability density function (PDF) and  $p(X|Y)$  is used to denote conditional probability density function of  $X$  given  $Y$ . Similarly,  $\Pr(\cdot)$  denotes the probability and  $\Pr(A|B)$  denotes the conditional probability of  $A$  given  $B$ . Symbol  $\approx$  denotes “approximately equal to”. The matrix  $\mathbf{0}_{N \times M}$  denotes a matrix of dimension  $N \times M$  with all zero entries and the matrix  $\mathbf{I}_N$  denotes an identity matrix of dimension  $N \times N$ . The function  $Q(\cdot)$  denotes the tail probability of the standard Normal distribution defined as  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du$ .

## II. SYSTEM DESCRIPTION

Consider a cooperative spectrum sensing scenario for primary user detection in a MIMO cognitive radio network. This can be modeled as a binary hypothesis testing problem with the null and alternative hypotheses  $\mathcal{H}_0$  and  $\mathcal{H}_1$  corresponding to the absence and presence of the primary user respectively. The cognitive radio network consists of a fusion center with  $N_f$  receive antennas and  $N$  cooperating secondary users with  $N_c$  antennas at each cooperating user. Let  $P_{D,i}$  and  $P_{F,i}$  denote the probabilities of detection and false alarm respectively of the local decision rule of the  $i$ th user. Once the  $i$ th secondary node has made a decision, it transmits a set of  $L$  vectors  $\mathbf{u}_i(l) \in \mathbb{C}^{N_c \times 1}$ ,  $1 \leq l \leq L$ , corresponding to its local decision

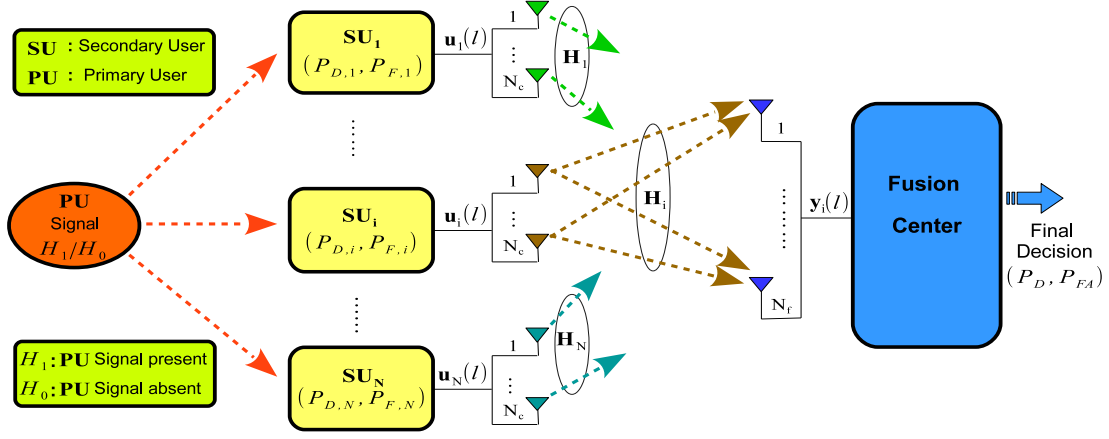


Fig. 1. System model of the cooperative MIMO cognitive radio sensor network consisting of  $N$  secondary users, each equipped with  $N_c$  antennas, communicating with a fusion center having  $N_f$  receive antennas.

regarding the absence or presence of the primary user, to the fusion center. Fig. 1 schematically illustrates the system under consideration. The matrix  $\mathbf{U}_i$  can be constructed by stacking the set of these  $L$  vectors  $\mathbf{u}_i(l)$ ,  $1 \leq l \leq L$  corresponding to the decision as  $\mathbf{U}_i = [\mathbf{u}_i(1), \mathbf{u}_i(2), \dots, \mathbf{u}_i(L)]^H \in \mathbb{C}^{L \times N_c}$ . Let the matrix  $\mathbf{U}_i$  transmitted to the fusion center take values  $\mathbf{U}_i = \mathbf{P}_0$  or  $\mathbf{U}_i = \mathbf{P}_1$  corresponding to the decisions of absence  $\mathcal{H}_0$  or presence  $\mathcal{H}_1$  of the primary user respectively. For instance, under non antipodal signalling with  $L = 2$  and  $N_c = 2$ , the matrix  $\mathbf{P}_0 = \mathbf{0}_{2 \times 2}$  and  $\mathbf{P}_1$  can be chosen as the orthogonal matrix  $[1 \ 1; 1 \ -1]_{2 \times 2}$ , i.e.,  $\mathbf{P}_1 \mathbf{P}_1^H = L \mathbf{I}_2$ . The local performance indices for the  $i$ th secondary user, i.e., probability of detection  $P_{D,i}$  and probability of false alarm  $P_{F,i}$ , can be expressed as,

$$P_{D,i} = \Pr(\mathbf{U}_i = \mathbf{P}_1 | \mathcal{H}_1), \quad (1)$$

$$1 - P_{D,i} = \Pr(\mathbf{U}_i = \mathbf{P}_0 | \mathcal{H}_1), \quad (2)$$

$$P_{F,i} = \Pr(\mathbf{U}_i = \mathbf{P}_1 | \mathcal{H}_0), \quad (3)$$

$$1 - P_{F,i} = \Pr(\mathbf{U}_i = \mathbf{P}_0 | \mathcal{H}_0). \quad (4)$$

Note that the number of antennas at the primary user and the detection scheme employed at the individual cooperating secondary users can be arbitrary. The received signal  $\mathbf{y}_i(l) \in \mathbb{C}^{N_f \times 1}$  at the fusion center corresponding to the vector  $\mathbf{u}_i(l)$  transmitted by the  $i$ th,  $1 \leq i \leq N$ , secondary user at the  $l$ th,  $1 \leq l \leq L$ , instant over an orthogonal MAC is given as,

$$\mathbf{y}_i(l) = \mathbf{H}_i \mathbf{u}_i(l) + \mathbf{w}_i(l), \quad (5)$$

where,  $\mathbf{H}_i \in \mathbb{C}^{N_f \times N_c}$  is the MIMO channel matrix with each element  $h_i(r, t)$  of the matrix  $\mathbf{H}_i$  representing the fading coefficient between the  $t$ th transmit antenna of the  $i$ th secondary user and the  $r$ th receive antenna of the fusion center. The  $j$ th row of the channel matrix  $\mathbf{H}_i = [\mathbf{h}_{i,1}, \mathbf{h}_{i,2}, \dots, \mathbf{h}_{i,N_f}]^H$  is given by  $\mathbf{h}_{i,j}^H \in \mathbb{C}^{1 \times N_c}$ . The vector  $\mathbf{w}_i(l) \in \mathbb{C}^{N_f \times 1}$  represents the circularly symmetric complex additive white Gaussian noise modeled as independent and identically distributed (i.i.d.) with zero mean and the covariance matrix given as  $\mathbf{R}_w = E\{\mathbf{w}_i(l) \mathbf{w}_i^H(l)\} = \sigma^2 \mathbf{I}_{N_f}$ . For simplicity of presentation, the received signal  $y_{i,j}(l)$  in (5) at the  $j$ th

receive antenna of the fusion center,  $1 \leq j \leq N_f$ , can be represented as,

$$y_{i,j}(l) = \mathbf{h}_{i,j}^H \mathbf{u}_i(l) + w_{i,j}(l), \quad (6)$$

where, the vector  $\mathbf{y}_i(l)$  in (5) is defined as  $\mathbf{y}_i(l) = [y_{i,1}(l), y_{i,2}(l), \dots, y_{i,N_f}(l)]^T \in \mathbb{C}^{N_f \times 1}$ . The term  $w_{i,j}(l)$  denotes the  $j$ th element of the noise vector  $\mathbf{w}_i(l)$ . The signal at the  $j$ th receive antenna of the fusion center for the set of  $L$  concatenated transmitted vectors corresponding to the decision of the  $i$ th secondary user, can be written as,

$$\mathbf{y}_{i,j} = \mathbf{U}_i \mathbf{h}_{i,j} + \mathbf{w}_{i,j}, \quad (7)$$

where,  $\mathbf{y}_{i,j} = [y_{i,j}(1), y_{i,j}(2), \dots, y_{i,j}(L)]^H \in \mathbb{C}^{L \times 1}$ . Similarly, the stacked noise vector  $\mathbf{w}_{i,j} \in \mathbb{C}^{L \times 1}$  is obtained as  $\mathbf{w}_{i,j} = [w_{i,j}(1), w_{i,j}(2), \dots, w_{i,j}(L)]^H$ . The next section presents the cooperative sensing schemes for MIMO cognitive radio networks under perfect CSI.

### III. COOPERATIVE MIMO SPECTRUM SENSING WITH PERFECT CSI

The received signal  $\mathbf{y}_i$  at the fusion center for the  $i$ th secondary user can be written as,  $\mathbf{y}_i = [\mathbf{y}_{i,1}^T, \mathbf{y}_{i,2}^T, \dots, \mathbf{y}_{i,N_f}^T]^T \in \mathbb{C}^{N_f L \times 1}$ . Stacking the received vectors  $\mathbf{y}_i$  for all the secondary users, the concatenated received signal  $\mathbf{y}$  is obtained as  $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_N^T]^T \in \mathbb{C}^{NN_f L \times 1}$ . Similarly, concatenating the MIMO channel matrices yields  $\mathbf{H} = [\mathbf{H}_1^T, \mathbf{H}_2^T, \dots, \mathbf{H}_N^T]^T \in \mathbb{C}^{NN_f \times N_c}$ . The LRT  $T(\mathbf{y})$  for this scenario based on the Neyman-Pearson (NP) criterion [30], which maximizes the probability of detection for a given probability of false alarm, can be expressed as,

$$T(\mathbf{y}) = \ln \left[ \frac{p(\mathbf{y}; \mathcal{H}_1)}{p(\mathbf{y}; \mathcal{H}_0)} \right] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma \quad (8)$$

where,  $p(\mathbf{y}; \mathcal{H}_0)$  and  $p(\mathbf{y}; \mathcal{H}_1)$  in (8) denote the PDFs of  $\mathbf{y}$  under the null and alternative hypotheses and  $\gamma$  denotes the detection threshold. The simplification in (11) follows from the independence of the decisions corresponding to the  $N$

$$T(\mathbf{y}) = \ln \left[ \prod_{i=1}^N \frac{p(\mathbf{y}_i; \mathcal{H}_1)}{p(\mathbf{y}_i; \mathcal{H}_0)} \right] \quad (11)$$

$$= \sum_{i=1}^N \ln \left[ \frac{p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_1) \Pr(\mathbf{U}_i = \mathbf{P}_1 | \mathcal{H}_1) + p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_0) \Pr(\mathbf{U}_i = \mathbf{P}_0 | \mathcal{H}_1)}{p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_1) \Pr(\mathbf{U}_i = \mathbf{P}_1 | \mathcal{H}_0) + p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_0) \Pr(\mathbf{U}_i = \mathbf{P}_0 | \mathcal{H}_0)} \right] \quad (12)$$

$$= \sum_{i=1}^N \ln \left[ \frac{P_{D,i} \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j} - \mathbf{P}_1 \mathbf{h}_{i,j}\|^2}{\sigma^2}\right) + (1 - P_{D,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j} - \mathbf{P}_0 \mathbf{h}_{i,j}\|^2}{\sigma^2}\right)}{P_{F,i} \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j} - \mathbf{P}_1 \mathbf{h}_{i,j}\|^2}{\sigma^2}\right) + (1 - P_{F,i}) \exp\left(-\sum_{j=1}^{N_f} \frac{\|\mathbf{y}_{i,j} - \mathbf{P}_0 \mathbf{h}_{i,j}\|^2}{\sigma^2}\right)} \right]. \quad (13)$$

secondary users using which the PDFs  $p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_1)$  and  $p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_0)$  can be written as,

$$p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_1) = \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}; \mathbf{U}_i = \mathbf{P}_1), \quad (9)$$

$$= \prod_{j=1}^{N_f} \frac{1}{\pi^L \sigma^{2L}} \exp \left[ \frac{-1}{\sigma^2} (\mathbf{y}_{i,j} - \mathbf{P}_1 \mathbf{h}_{i,j})^H \mathbf{I}_L^{-1} (\mathbf{y}_{i,j} - \mathbf{P}_1 \mathbf{h}_{i,j}) \right],$$

$$p(\mathbf{y}_i; \mathbf{U}_i = \mathbf{P}_0) = \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}; \mathbf{U}_i = \mathbf{P}_0), \quad (10)$$

$$= \prod_{j=1}^{N_f} \frac{1}{\pi^L \sigma^{2L}} \exp \left[ \frac{-1}{\sigma^2} (\mathbf{y}_{i,j} - \mathbf{P}_0 \mathbf{h}_{i,j})^H \mathbf{I}_L^{-1} (\mathbf{y}_{i,j} - \mathbf{P}_0 \mathbf{h}_{i,j}) \right],$$

where the individual PDFs  $p(\mathbf{y}_{i,j}; \mathbf{U}_i = \mathbf{P}_1)$  and  $p(\mathbf{y}_{i,j}; \mathbf{U}_i = \mathbf{P}_0)$  in (9) and (10) correspond to the complex Normal distributions  $\mathcal{CN}(\mathbf{P}_1 \mathbf{h}_{i,j}, \sigma^2 \mathbf{I}_L)$  and  $\mathcal{CN}(\mathbf{P}_0 \mathbf{h}_{i,j}, \sigma^2 \mathbf{I}_L)$ , respectively. Substituting these PDFs in (12), the likelihood ratio test statistic  $T(\mathbf{y})$  for the multiuser cooperative MIMO cognitive radio sensing scenario under consideration can be obtained as (13). Depending on the signaling scheme employed, the set of  $L$  vectors transmitted by the  $i$ th secondary user, i.e.,  $\mathbf{u}_i(l) \in \mathbb{C}^{N_c \times 1}, 1 \leq l \leq L$  and  $\mathbf{U}_i = [\mathbf{u}_i(1), \dots, \mathbf{u}_i(L)]^H \in \mathbb{C}^{L \times N_c}$ , with  $\mathbf{U}_i \in \{\mathbf{P}_0, \mathbf{P}_1\}$  corresponding to its local decision can either be antipodal or non-antipodal in nature. The test statistic in (13) is further simplified next corresponding to these different signalling formats.

#### A. Antipodal Signaling

Consider antipodal signaling where the  $i$ th secondary user transmits  $\mathbf{U}_i = -\mathbf{P}$  or  $\mathbf{U}_i = \mathbf{P}$ , corresponding to the local decisions regarding the absence or presence of the primary signal. A simple and efficient choice for the decision matrix  $\mathbf{P}$  is an orthogonal matrix satisfying  $\mathbf{P}^H \mathbf{P} \propto \mathbf{I}$ . Further, if the total energy is  $E$ , one can choose  $\mathbf{P}$  such that  $\text{Trace}(\mathbf{P}^H \mathbf{P}) = E \implies \mathbf{P}^H \mathbf{P} = \frac{E}{N_c} \mathbf{I}$ . The test statistic in (13) can be simplified for this signaling scheme as,

$$T_A(\mathbf{y}) = \sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \exp\left(-\frac{4}{\sigma^2} \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j})\right)}{P_{F,i} + (1 - P_{F,i}) \exp\left(-\frac{4}{\sigma^2} \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j})\right)} \right]. \quad (14)$$

Employing the approximations  $e^{-t} \approx (1 - t)$ ,  $\ln(1 + t) \approx t$ , for small values of  $t$ , the test statistic  $T_A(\mathbf{y})$  in (14) can be further simplified at low SNR as,

$$T_A(\mathbf{y}) = \sum_{i=1}^N \left[ a_i \underbrace{\sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j})}_{\mathbb{T}(\mathbf{y}_i)} \right], \quad (15)$$

where the symbol  $\Re(\cdot)$  denotes the real part, the scaling constant  $a_i$  for the  $i$ th secondary user is defined as  $a_i = P_{D,i} - P_{F,i}$  and the term  $\mathbb{T}(\mathbf{y}_i)$  of the test statistic  $T_A(\mathbf{y})$  is defined as  $\mathbb{T}(\mathbf{y}_i) = a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j})$ . For the special case with identical local detection performance for all the cooperating users, i.e.,  $P_{D,i} = P_d$  and  $P_{F,i} = P_f$ , the test statistic in (15) reduces to,

$$T_{A-I}(\mathbf{y}) = \sum_{i=1}^N \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}). \quad (16)$$

Note that the low SNR approximation employed above is required only to further simplify the NP criterion based likelihood ratio test given in (14) to obtain the simplified tests in (15) and (16). At high SNR, one can directly employ the NP test given in (14). More importantly, the low SNR regime is rather critical and relevant for spectrum sensing in cognitive radio scenarios [31], [32], since the primary user SNR can be as low as  $-10$  dB to  $-25$  dB as per FCC regulations [33]–[35]. Thus, the design and analysis of multi-user cooperative MIMO detection schemes, which are especially suited for spectrum sensing in the low SNR regime, are relevant for practical cognitive radio deployments. The next result characterizes the performance of the detector in (15) for cooperative MIMO spectrum sensing.

**Theorem 1.** *For a given threshold  $\gamma$ , the probabilities of false alarm ( $P_{FA}$ ) and detection ( $P_D$ ) at the fusion center for the detector based on the test statistic in (15) towards cooperative spectrum sensing in MIMO cognitive radio networks under antipodal signaling, are given as,*

$$P_{FA} = Q\left(\frac{\gamma - \mu_{T_A|\mathcal{H}_0}}{\sigma_{T_A|\mathcal{H}_0}}\right), \quad (17)$$

$$P_D = Q\left(\frac{\gamma - \mu_{T_A|\mathcal{H}_1}}{\sigma_{T_A|\mathcal{H}_1}}\right), \quad (18)$$

where,  $\mu_{T_A|\mathcal{H}_0}$ ,  $\mu_{T_A|\mathcal{H}_1}$ ,  $\sigma_{T_A|\mathcal{H}_0}^2$ ,  $\sigma_{T_A|\mathcal{H}_1}^2$  denote the means and the variances of the test statistic  $T_A(\mathbf{y})$  in (15) respectively

$$T_N(\mathbf{y}) = \sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \exp \left( -\frac{1}{\sigma^2} \sum_{j=1}^{N_f} \left( \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}) - \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right) \right)}{P_{F,i} + (1 - P_{F,i}) \exp \left( -\frac{1}{\sigma^2} \sum_{j=1}^{N_f} \left( \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}) - \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right) \right)} \right] \quad (21)$$

$$\approx \sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \left( 1 - \frac{1}{\sigma^2} \sum_{j=1}^{N_f} \left( \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}) - \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right) \right)}{P_{F,i} + (1 - P_{F,i}) \left( 1 - \frac{1}{\sigma^2} \sum_{j=1}^{N_f} \left( \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}) - \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right) \right)} \right]. \quad (22)$$

corresponding to the two hypotheses. These are given for  $\mathcal{H}_1$  as,

$$\mu_{T_A|\mathcal{H}_1} = \sum_{i=1}^N \left[ a_i b_i \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right], \quad (19)$$

$$\sigma_{T_A|\mathcal{H}_1}^2 = \sum_{i=1}^N a_i^2 \left[ b_i^2 \sum_{j=1}^{N_f} \sum_{k=1, k \neq j}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \|\mathbf{P} \mathbf{h}_{i,k}\|^2 + \sum_{j=1}^{N_f} \left[ \|\mathbf{P} \mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right] - \left[ b_i \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right]^2 \right], \quad (20)$$

where,  $b_i$  is given as  $b_i = (2P_{D,i} - 1)$ . Similarly, the mean  $\mu_{T_A|\mathcal{H}_0}$  and the variance  $\sigma_{T_A|\mathcal{H}_0}^2$  corresponding to the null hypothesis  $\mathcal{H}_0$  can be obtained by replacing  $b_i$  with  $c_i$ ,  $c_i = (2P_{F,i} - 1)$ , in the expressions for  $\mu_{T_A|\mathcal{H}_1}$  in (19) and  $\sigma_{T_A|\mathcal{H}_1}^2$  in (20) respectively.

*Proof.* See Appendix A.  $\square$

A similar result has been derived in [29] for decision fusion with MRC in a coherent multiple access channel with multiple single antenna sensor nodes and a massive antenna array at the fusion center. A framework to derive the optimal decision matrix based on maximizing the deflection coefficient for MIMO spectrum sensing has been presented in [27]. A similar framework can be explored in the context of cooperative MIMO spectrum sensing scenarios considered in this work.

### B. Non-Antipodal Signaling

The test statistic under non-antipodal signaling, i.e., with the transmitted matrix  $\mathbf{U}_i = \mathbf{P}$  or  $\mathbf{0}_{L \times N_c}$  in (13), for the local decision corresponding to the presence or absence of primary user signal respectively, is given in (21), where (22) follows from a low SNR approximation of  $e^{-x}$  as  $e^{-x} \approx 1 - x$  for small values of  $x$ . Further using the approximation  $\ln(1+x) \approx x$  for small values of  $x$ , the cooperative sensing statistic  $T_N(\mathbf{y})$  can be simplified as,

$$T_N(\mathbf{y}) = \sum_{i=1}^N \underbrace{\left[ a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}) \right]}_{\mathcal{T}(\mathbf{y}_i)}, \quad (23)$$

where  $a_i = P_{D,i} - P_{F,i}$ ,  $T_N(\mathbf{y}) = \sum_{i=1}^N \mathcal{T}(\mathbf{y}_i)$  with  $\mathcal{T}(\mathbf{y}_i) = a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j})$  and the constant terms that are independent of  $\mathbf{y}$  are merged with the threshold  $\gamma$ . For the

special case of non-antipodal signaling under identical local detection performance of all secondary users, i.e.,  $P_{D,i} = P_d$  and  $P_{F,i} = P_f$ ,  $\forall i$ , the test statistic in (23) further reduces to,

$$T_{N-1}(\mathbf{y}) = \sum_{i=1}^N \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}). \quad (24)$$

The analytical expressions for the detection performance when using the test statistic in (23) are derived below.

**Theorem 2.** *The probabilities of false alarm and detection  $P_{FA}$  and  $P_D$  respectively at the fusion center for detection based on test statistic in (23) towards cooperative spectrum sensing are given as,*

$$P_{FA} = Q \left( \frac{\gamma - \mu_{T_N|\mathcal{H}_0}}{\sigma_{T_N|\mathcal{H}_0}} \right), \quad (25)$$

$$P_D = Q \left( \frac{\gamma - \mu_{T_N|\mathcal{H}_1}}{\sigma_{T_N|\mathcal{H}_1}} \right), \quad (26)$$

where  $\mu_{T_N|\mathcal{H}_0}$ ,  $\mu_{T_N|\mathcal{H}_1}$  and  $\sigma_{T_N|\mathcal{H}_0}^2$ ,  $\sigma_{T_N|\mathcal{H}_1}^2$ , denote the means and variances of the test statistic  $T_N(\mathbf{y})$  in (23) under the null and alternative hypotheses  $\mathcal{H}_0$ ,  $\mathcal{H}_1$ , respectively. The mean and variance under the alternative hypothesis  $\mathcal{H}_1$  are,

$$\mu_{T_N|\mathcal{H}_1} = \sum_{i=1}^N \left[ a_i P_{D,i} \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right], \quad (27)$$

$$\sigma_{T_N|\mathcal{H}_1}^2 = \sum_{i=1}^N a_i^2 \left[ P_{D,i}^2 \sum_{j=1}^{N_f} \sum_{k=1, k \neq j}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \|\mathbf{P} \mathbf{h}_{i,k}\|^2 + \sum_{j=1}^{N_f} \left[ P_{D,i}^2 \|\mathbf{P} \mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right] - \left[ P_{D,i} \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right]^2 \right]. \quad (28)$$

The expressions for the mean  $\mu_{T_N|\mathcal{H}_0}$  in (94) and the variance  $\sigma_{T_N|\mathcal{H}_0}^2$  in (101) corresponding to the null hypothesis can be obtained by replacing  $P_{D,i}$  with  $P_{F,i}$  in the expressions of  $\mu_{T_N|\mathcal{H}_1}$  in (27) and  $\sigma_{T_N|\mathcal{H}_1}^2$  in (28) respectively.

*Proof.* See Appendix B.  $\square$

It is important to note at this juncture that the performance of the detectors  $T_A(\mathbf{y})$  in (15) and  $T_N(\mathbf{y})$  in (23) depend critically upon the accuracy of the available CSI. The time selective fading nature of the wireless links between the secondary users and the fusion center makes accurate estimation of the fading channel coefficients a challenging task. Thus, in practice, one can only obtain CSI with a limited degree of accuracy. Therefore, the subsequent sections derive robust cooperative schemes for spectrum sensing in the presence of CSI uncertainty.

$$T_R(\mathbf{y}) = \ln \left[ \prod_{i=1}^N \frac{p(\mathbf{y}_i|\mathcal{H}_1; \hat{\mathbf{H}}_i)}{p(\mathbf{y}_i|\mathcal{H}_0; \hat{\mathbf{H}}_i)} \right] \quad (38)$$

$$= \sum_{i=1}^N \ln \left[ \frac{(P_{D,i})p(\mathbf{y}_i|\mathbf{U}_i = \mathbf{P}_1; \hat{\mathbf{H}}_i) + (1 - P_{D,i})p(\mathbf{y}_i|\mathbf{U}_i = \mathbf{P}_0; \hat{\mathbf{H}}_i)}{(P_{F,i})p(\mathbf{y}_i|\mathbf{X}_k = \mathbf{X}_1; \hat{\mathbf{H}}_i) + (1 - P_{F,i})p(\mathbf{y}_i|\mathbf{U}_i = \mathbf{P}_0; \hat{\mathbf{H}}_i)} \right] \quad (39)$$

$$= \sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \exp \left( - \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}) \right)}{P_{F,i} + (1 - P_{F,i}) \exp \left( - \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}) \right)} \right]. \quad (40)$$

#### IV. ROBUST COOPERATIVE SPECTRUM SENSING UNDER CSI UNCERTAINTY

In the presence of CSI uncertainty [27], the true channel matrix  $\mathbf{H}_i$  can be expressed as,

$$\mathbf{H}_i = \hat{\mathbf{H}}_i + \mathbf{E}_i, \quad (29)$$

where,  $\hat{\mathbf{H}}_i \in \mathbb{C}^{N_f \times N_c}$  denotes the available imperfect estimate of the MIMO channel matrix for the  $i^{\text{th}}$ ,  $1 \leq i \leq N$ , secondary user. The matrix  $\mathbf{E}_i = [\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \dots, \mathbf{e}_{i,N_f}]^H \in \mathbb{C}^{N_f \times N_c}$  denotes the uncertainty in the obtained estimate of  $\mathbf{H}_i$ . This is similar to the CSI uncertainty model employed in works such as [29]. Let  $\mathbf{e}_{i,j}^H \in \mathbb{C}^{1 \times N_c}$  represent the  $j^{\text{th}}$ ,  $1 \leq j \leq N_f$ , row of the uncertainty matrix  $\mathbf{E}_i$  that is assumed to follow a complex Gaussian distribution with zero mean and covariance matrix  $\mathbf{R}_{e,i} = E\{\mathbf{e}_{i,j} \mathbf{e}_{i,j}^H\}$ , i.e.,  $\mathbf{e}_{i,j} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{e,i})$ . The received signal  $\mathbf{y}_i(l) \in \mathbb{C}^{N_f \times 1}$  in (5) at the fusion center for the  $i^{\text{th}}$ ,  $1 \leq i \leq N$ , secondary user corresponding to the  $l^{\text{th}}$ ,  $1 \leq l \leq L$ , transmitted vector can be expressed as,

$$\mathbf{y}_i(l) = (\hat{\mathbf{H}}_i + \mathbf{E}_i) \mathbf{u}_i(l) + \mathbf{w}_i(l). \quad (30)$$

The received signal component  $y_{i,j}(l)$  at the  $j^{\text{th}}$ ,  $1 \leq j \leq N_f$ , receive antenna of the fusion center can be expressed as,

$$y_{i,j}(l) = (\hat{\mathbf{h}}_{i,j}^H + \mathbf{e}_{i,j}^H) \mathbf{u}_i(l) + w_{i,j}(l), \quad (31)$$

where,  $\hat{\mathbf{h}}_{i,j}^H$  denotes the  $j^{\text{th}}$  row of the estimate of the channel estimate matrix  $\hat{\mathbf{H}}_i = [\hat{\mathbf{h}}_{i,1}, \hat{\mathbf{h}}_{i,2}, \dots, \hat{\mathbf{h}}_{i,N_f}]^H$ . Similar to the perfect CSI scenario, assume that the  $i^{\text{th}}$  secondary user transmits a set of  $L$  vectors  $\mathbf{u}_i(l)$ ,  $1 \leq l \leq L$ , with the transmitted matrix  $\mathbf{U}_i \in \mathbb{C}^{L \times N_c}$  defined as  $\mathbf{U}_i = [\mathbf{u}_i(1), \mathbf{u}_i(2), \dots, \mathbf{u}_i(L)]^H$  corresponding to the local decision indicating the absence or the presence of the primary user signal respectively. Hence, the concatenated received signal  $\mathbf{y}_{i,j} \in \mathbb{C}^{L \times 1}$  for the set of  $L$  vectors corresponding to the local decision of the  $i^{\text{th}}$  secondary user at the  $j^{\text{th}}$  receive antenna of the fusion center can be expressed as,

$$\mathbf{y}_{i,j} = \mathbf{U}_i (\hat{\mathbf{h}}_{i,j} + \mathbf{e}_{i,j}) + \mathbf{w}_{i,j}, \quad (32)$$

where,  $\mathbf{y}_{i,j} = [y_{i,j}(1), \dots, y_{i,j}(L)]^H \in \mathbb{C}^{L \times 1}$ . The noise vector  $\mathbf{w}_{i,j} = [w_{i,j}(1), \dots, w_{i,j}(L)]^H \in \mathbb{C}^{L \times 1}$  is the stacked complex Gaussian noise vector with zero mean and the covariance matrix  $\sigma^2 \mathbf{I}_L$ . Consider antipodal signaling in which the  $i^{\text{th}}$  secondary user transmits a set of  $L$  antipodal vectors to the fusion center for the local decision, i.e.,  $\mathbf{U}_i = \mathbf{P}$  or  $\mathbf{U}_i = -\mathbf{P}$

indicating the presence or absence of the primary user signal respectively, where  $\mathbf{U}_i = [\mathbf{u}_i(1), \mathbf{u}_i(2), \dots, \mathbf{u}_i(L)]^H$ . The PDFs of the received vector  $\mathbf{y}_{i,j}$  in (32) corresponding to the local decisions of the secondary users can be obtained as,

$$\begin{aligned} p(\mathbf{y}_{i,j}|\mathbf{U}_i = +\mathbf{P}; \hat{\mathbf{h}}_{i,j}) &\sim \mathcal{CN}(+\mathbf{P}\hat{\mathbf{h}}_{i,j}, \Gamma_i) \\ p(\mathbf{y}_{i,j}|\mathbf{U}_i = -\mathbf{P}; \hat{\mathbf{h}}_{i,j}) &\sim \mathcal{CN}(-\mathbf{P}\hat{\mathbf{h}}_{i,j}, \Gamma_i), \end{aligned} \quad (33)$$

where the covariance matrix of  $\mathbf{y}_{i,j}$  is

$$\Gamma_i = E\{(\mathbf{U}_i \mathbf{e}_{i,j} + \mathbf{w}_{i,j})(\mathbf{U}_i \mathbf{e}_{i,j} + \mathbf{w}_{i,j})^H\} \quad (34)$$

$$= E\{\mathbf{U}_i \mathbf{e}_{i,j} \mathbf{e}_{i,j}^H \mathbf{U}_i\} + E\{\mathbf{w}_{i,j} \mathbf{w}_{i,j}^H\} \quad (35)$$

$$= \mathbf{P} \mathbf{R}_{e,i} \mathbf{P}^H + \sigma^2 \mathbf{I}_L. \quad (36)$$

Let  $\Gamma_i = \mathbf{Q}_i \mathbf{Q}_i^H$  denote the standard Cholesky decomposition [36] of the positive semi-definite matrix  $\Gamma_i$ . Let the concatenated receive vector  $\mathbf{y}$  be stacked as  $\mathbf{y} = [\mathbf{y}_1^T, \mathbf{y}_2^T, \dots, \mathbf{y}_N^T]^T \in \mathbb{C}^{NN_f L \times 1}$  corresponding to the vectors  $\mathbf{u}_i(l)$ ,  $1 \leq l \leq L$  for the  $N$  secondary users at the  $N_f$  receive antennas of the fusion center, where each component  $\mathbf{y}_i$ ,  $1 \leq i \leq N$  is given as  $\mathbf{y}_i = [\mathbf{y}_{i,1}^T, \mathbf{y}_{i,2}^T, \dots, \mathbf{y}_{i,N_f}^T]^T \in \mathbb{C}^{N_f L \times 1}$ . Similarly, let the estimates of the MIMO channel matrices corresponding to the  $N$  secondary users be stacked as  $\hat{\mathbf{H}} = [\hat{\mathbf{H}}_1^T, \hat{\mathbf{H}}_2^T, \dots, \hat{\mathbf{H}}_N^T]^T \in \mathbb{C}^{NN_f \times N_c}$ . The uncertainty statistics based likelihood ratio test (US-LRT) statistic  $T_R(\mathbf{y})$  and the associated test for robust cooperative spectrum sensing now follows employing the Neyman-Pearson (NP) criterion [30] as,

$$T_R(\mathbf{y}) = \ln \left[ \frac{p(\mathbf{y}|\mathcal{H}_1; \hat{\mathbf{H}})}{p(\mathbf{y}|\mathcal{H}_0; \hat{\mathbf{H}})} \right] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \gamma, \quad (37)$$

where,  $p(\mathbf{y}|\mathcal{H}_0; \hat{\mathbf{H}})$  in (37) and  $p(\mathbf{y}_i|\mathcal{H}_0; \hat{\mathbf{H}}_i)$  in (38) denote the PDFs of  $\mathbf{y}$  and  $\mathbf{y}_i$  corresponding to the nominal estimates  $\hat{\mathbf{H}}$  and  $\hat{\mathbf{H}}_i$  of the true channel matrix  $\mathbf{H}$  and  $\mathbf{H}_i$  respectively under the null hypothesis  $\mathcal{H}_0$ . Similarly,  $p(\mathbf{y}, \hat{\mathbf{H}}|\mathcal{H}_1)$  in (37) and  $p(\mathbf{y}_i, \hat{\mathbf{H}}_i|\mathcal{H}_1)$  in (38) denote the PDFs corresponding to the alternative hypothesis  $\mathcal{H}_1$ . The test in (37) can be expressed as the test statistic  $T_R(\mathbf{y})$  in (40), where (39) follows from the independence of the receive vectors of the different secondary users given the nominal estimates  $\hat{\mathbf{H}}_i$ . Using the independence of the signals received at the antennas of the fusion center, it follows that  $p(\mathbf{y}_i|\mathbf{U}_i = \pm\mathbf{P}; \hat{\mathbf{H}}_i) = \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}|\mathbf{U}_i = \pm\mathbf{P}; \hat{\mathbf{H}}_i)$ . Subsequently employing the PDFs for the various quantities yields

the expression in (40). At low SNR, the test statistic  $T_R(\mathbf{y})$  can be simplified as,

$$T_R(\mathbf{y}) = \sum_{i=1}^N \underbrace{\left[ a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}) \right]}_{T(\mathbf{y}_i)}, \quad (41)$$

where  $T(\mathbf{y}_i)$  is defined as,  $T_R(\mathbf{y}) = \sum_{i=1}^N T_i(\mathbf{y})$ . Consider now a special case in which all the cooperating secondary users have identical local detection performance, i.e.,  $P_{D,i} = P_d$  and  $P_{F,i} = P_f, \forall i$ . In this case, the robust detector for the cooperative multiuser MIMO cognitive radio system, reduces to,

$$T_{R-I}(\mathbf{y}) = \sum_{i=1}^N \left[ \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}) \right]. \quad (42)$$

Further observe that the test statistic in (41) is a linear combination of weighted Normal random variables and hence follows a Normal distribution. Theorem 3 below derives the requisite analytical expressions to characterize its detection performance.

**Theorem 3.** *The probabilities of false alarm  $P_{FA}$  and detection  $P_D$  for a given threshold  $\gamma$  at the fusion center for the test statistic  $T_R(\mathbf{y})$  in (41) for robust cooperative spectrum sensing in MIMO cognitive radio networks are,*

$$P_{FA} = Q \left( \frac{\gamma - \mu_{T_R|\mathcal{H}_0}}{\sigma_{T_R|\mathcal{H}_0}} \right), \quad (43)$$

$$P_D = Q \left( \frac{\gamma - \mu_{T_R|\mathcal{H}_1}}{\sigma_{T_R|\mathcal{H}_1}} \right), \quad (44)$$

where,  $\mu_{T_R|\mathcal{H}_0}/\mu_{T_R|\mathcal{H}_1}$  and  $\sigma_{T_R|\mathcal{H}_0}^2/\sigma_{T_R|\mathcal{H}_1}^2$  denote the means and the variances of the test statistic  $T_R(\mathbf{y})$  under the null and alternative hypotheses respectively. The mean and the variance of  $T_R(\mathbf{y})$  corresponding to the alternative hypothesis are,

$$\mu_{T_R|\mathcal{H}_1} = \sum_{i=1}^N \left[ a_i b_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right], \quad (45)$$

$$\begin{aligned} \sigma_{T_R|\mathcal{H}_1}^2 &= \sum_{i=1}^N a_i^2 \left[ b_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,k}\|^2 \right. \\ &+ \sum_{j=1}^{N_f} \left[ \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^4 + \frac{1}{2} \|\mathbf{C}_i^{-1} \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right. \\ &\left. \left. + \frac{\sigma^2}{2} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right] - \left[ b_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \mathbf{h}_{i,j}\|^2 \right]^2 \right], \quad (46) \end{aligned}$$

where,  $\Gamma_i = \mathbf{Q}_i \mathbf{Q}_i^H$ ,  $\mathbf{R}_{u,i} = \mathbf{C}_i \mathbf{C}_i^H$  and  $b_i = 2P_{D,i} - 1$ . Similarly, the expressions for the mean  $\mu_{T_R|\mathcal{H}_0}$  and the variance

$\sigma_{T_R|\mathcal{H}_0}^2$  corresponding to the null hypothesis  $\mathcal{H}_0$  are given as,

$$\mu_{T_R|\mathcal{H}_0} = \sum_{i=1}^N \left[ a_i c_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right], \quad (47)$$

$$\begin{aligned} \sigma_{T_R|\mathcal{H}_0}^2 &= \sum_{i=1}^N a_i^2 \left[ c_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,k}\|^2 \right. \\ &+ \sum_{j=1}^{N_f} \left[ \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^4 + \frac{1}{2} \|\mathbf{C}_i^{-1} \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right. \\ &\left. \left. + \frac{\sigma^2}{2} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right] - \left[ c_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \mathbf{h}_{i,j}\|^2 \right]^2 \right], \quad (48) \end{aligned}$$

where,  $c_i = 2P_{F,i} - 1$ .

It is evident that the matrix  $\mathbf{Q}_i$  obtained from the Cholesky decomposition of the noise plus uncertainty covariance  $\Gamma_i$  plays a key role in the expressions obtained for the various quantities above in contrast to those obtained for the scenario with perfect CSI availability in Theorems 1 and 2. Further, it can also be seen that the channel vectors  $\mathbf{h}_{i,j}$  are naturally replaced by their nominal estimates  $\hat{\mathbf{h}}_{i,j}$ .

*Proof.* See Appendix C.  $\square$

The generalized likelihood ratio test (GLRT) based robust detector for cooperative spectrum sensing is developed next.

## V. ROBUST GENERALIZED LIKELIHOOD RATIO TEST (RGLRT)

The GLRT framework enables the development of detection schemes in the presence of unknown parameters. This property of the GLRT is leveraged in this section to propose spectrum sensing schemes that are robust under CSI uncertainty. The binary hypothesis testing problem for concatenated received vector  $\mathbf{y}_{i,j}$  in (32) with CSI uncertainty can be formulated as,

$$\mathcal{H}_0 : \mathbf{y}_{i,j} = \mathbf{P}_0 \mathbf{h}_{i,j} + \mathbf{w}_{i,j} = \mathbf{P}_0 (\hat{\mathbf{h}}_{i,j} + \mathbf{e}_{i,j}) + \mathbf{w}_{i,j} \quad (49)$$

$$\mathcal{H}_1 : \mathbf{y}_{i,j} = \mathbf{P}_1 \mathbf{h}_{i,j} + \mathbf{w}_{i,j} = \mathbf{P}_1 (\hat{\mathbf{h}}_{i,j} + \mathbf{e}_{i,j}) + \mathbf{w}_{i,j} \quad (50)$$

Let the vectors  $\mathbf{v}_{i,j|0}, \mathbf{v}_{i,j|1}$  for the hypotheses  $\mathcal{H}_1$  and  $\mathcal{H}_0$  be defined as,

$$\mathbf{v}_{i,j|0} = \mathbf{y}_{i,j} - \mathbf{P}_0 \hat{\mathbf{h}}_{i,j} = \mathbf{P}_0 \mathbf{e}_{i,j} + \mathbf{w}_{i,j} = \underbrace{[\mathbf{P}_0 \ \mathbf{I}_L]}_{\mathbf{A}_{i|0}} \underbrace{\begin{bmatrix} \mathbf{e}_{i,j} \\ \mathbf{w}_{i,j} \end{bmatrix}}_{\mathbf{z}_{i,j}} \quad (51)$$

$$\mathbf{v}_{i,j|1} = \mathbf{y}_{i,j} - \mathbf{P}_1 \hat{\mathbf{h}}_{i,j} = \mathbf{P}_1 \mathbf{e}_{i,j} + \mathbf{w}_{i,j} = \underbrace{[\mathbf{P}_1 \ \mathbf{I}_L]}_{\mathbf{A}_{i|1}} \underbrace{\begin{bmatrix} \mathbf{e}_{i,j} \\ \mathbf{w}_{i,j} \end{bmatrix}}_{\mathbf{z}_{i,j}} \quad (52)$$

where the matrix  $\mathbf{A}_i = [\mathbf{U}_i \ \mathbf{I}_L] \in \mathbb{C}^{L \times (N_c + L)}$  is obtained by augmenting the local decision matrix  $\mathbf{U}_i \in \{\mathbf{P}_0, \mathbf{P}_1\}$  transmitted by the secondary user and the identity matrix  $\mathbf{I}_L$ . The augmented matrices  $\mathbf{A}_i$  corresponding to the presence, absence of the primary user signal are  $\mathbf{A}_{i|1} = [\mathbf{P}_1 \ \mathbf{I}_L]$ ,  $\mathbf{A}_{i|0} = [\mathbf{P}_0 \ \mathbf{I}_L]$  respectively. The concatenated vector  $\mathbf{z}_{i,j} \in \mathbb{C}^{(N_c + L) \times 1}$  is the unknown vector defined as  $\mathbf{z}_{i,j} = [\mathbf{e}_{i,j}^T, \mathbf{w}_{i,j}^T]^T$ .

$$T_{\text{N-RGLRT}}(\mathbf{y}) = \sum_{i=1}^N \ln \left[ \frac{P_{D,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1, \hat{\mathbf{e}}_{i,j|1}) + (1 - P_{D,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j} | \mathbf{U}_i = \mathbf{0}; \mathcal{H}_1)}{P_{F,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_0, \hat{\mathbf{e}}_{i,j|1}) + (1 - P_{F,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j} | \mathbf{U}_i = \mathbf{0}; \mathcal{H}_0)} \right] \quad (60)$$

$$= \sum_{i=1}^N \ln \left[ \frac{P_{D,i} \prod_{j=1}^{N_f} \frac{1}{\pi^L \sigma^{2L}} \exp \left[ -\frac{1}{\sigma^2} \mathbf{y}_{i,j}^H \mathcal{P}_{\mathbf{X}}^\perp \mathbf{y}_{i,j} \right] + (1 - P_{D,i}) \prod_{j=1}^{N_f} \frac{1}{\pi^L \sigma^{2L}} \exp \left[ -\frac{1}{\sigma^2} \mathbf{y}_{i,j}^H \mathbf{y}_{i,j} \right]}{P_{F,i} \prod_{j=1}^{N_f} \frac{1}{\pi^L \sigma^{2L}} \exp \left[ -\frac{1}{\sigma^2} \mathbf{y}_{i,j}^H \mathcal{P}_{\mathbf{X}}^\perp \mathbf{y}_{i,j} \right] + (1 - P_{F,i}) \prod_{j=1}^{N_f} \frac{1}{\pi^L \sigma^{2L}} \exp \left[ -\frac{1}{\sigma^2} \mathbf{y}_{i,j}^H \mathbf{y}_{i,j} \right]} \right] \quad (61)$$

$$= \sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \prod_{j=1}^{N_f} \exp \left[ -\mathbf{y}_{i,j}^H (\mathbf{I} - \mathcal{P}_{\mathbf{X}}^\perp) \mathbf{y}_{i,j} \right]}{P_{F,i} + (1 - P_{F,i}) \prod_{j=1}^{N_f} \exp \left[ -\mathbf{y}_{i,j}^H (\mathbf{I} - \mathcal{P}_{\mathbf{X}}^\perp) \mathbf{y}_{i,j} \right]} \right] = \sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \exp \left[ -\sum_{j=1}^{N_f} \mathbf{y}_{i,j}^H \mathcal{P}_{\mathbf{X}} \mathbf{y}_{i,j} \right]}{P_{F,i} + (1 - P_{F,i}) \exp \left[ -\sum_{j=1}^{N_f} \mathbf{y}_{i,j}^H \mathcal{P}_{\mathbf{X}} \mathbf{y}_{i,j} \right]} \right]. \quad (62)$$

### A. Non-antipodal signaling

The non-antipodal signaling matrices  $\mathbf{P}_1 = \mathbf{P}$  and  $\mathbf{P}_0 = \mathbf{0}_{L \times N_c}$  are matrices corresponding to the local decisions for the presence and absence of the primary signal respectively. Under the alternative hypothesis, the likelihood of the received vector  $\mathbf{y}_{i,j}$  parameterized by  $\mathbf{e}_{i,j}$  can be obtained as,

$$\begin{aligned} p(\mathbf{y}_{i,j} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1, \mathbf{e}_{i,j}) \\ = \frac{1}{\pi^L \sigma^{2L}} \exp \left[ -\frac{1}{\sigma^2} (\mathbf{v}_{i,j|1} - \mathbf{P} \mathbf{e}_{i,j})^H (\mathbf{v}_{i,j|1} - \mathbf{P} \mathbf{e}_{i,j}) \right]. \end{aligned} \quad (53)$$

The MLE of the uncertainty vector  $\mathbf{e}_{i,j}$ , under alternative hypothesis  $\mathcal{H}_1$ , is given as  $\hat{\mathbf{e}}_{i,j|1} = (\mathbf{P}^H \mathbf{P})^{-1} \mathbf{P}^H \mathbf{v}_{i,j|1}$ . Using the MLE  $\hat{\mathbf{e}}_{i,j|1}$ , the argument of the exponential term in (53) can be simplified as,

$$\begin{aligned} (\mathbf{v}_{i,j|1} - \mathbf{P} \hat{\mathbf{e}}_{i,j|1})^H (\mathbf{v}_{i,j|1} - \mathbf{P} \hat{\mathbf{e}}_{i,j|1}) \\ = \mathbf{v}_{i,j|1}^H (\mathbf{I} - \mathbf{P} (\mathbf{P}^H \mathbf{P})^{-1} \mathbf{P}^H)^H (\mathbf{I} - \mathbf{P} (\mathbf{P}^H \mathbf{P})^{-1} \mathbf{P}^H) \mathbf{v}_{i,j|1} \end{aligned} \quad (54)$$

$$= \mathbf{v}_{i,j|1}^H \mathcal{P}_{\mathbf{P}}^\perp \mathcal{P}_{\mathbf{P}}^\perp \mathbf{v}_{i,j|1} \quad (55)$$

$$= (\mathbf{y}_{i,j} - \mathbf{P} \hat{\mathbf{h}}_{i,j})^H \mathcal{P}_{\mathbf{P}}^\perp \mathcal{P}_{\mathbf{P}}^\perp (\mathbf{y}_{i,j} - \mathbf{P} \hat{\mathbf{h}}_{i,j}) \quad (56)$$

$$= (\mathcal{P}_{\mathbf{P}}^\perp \mathbf{y}_{i,j} - \mathcal{P}_{\mathbf{P}}^\perp \mathbf{P} \hat{\mathbf{h}}_{i,j})^H (\mathcal{P}_{\mathbf{P}}^\perp \mathbf{y}_{i,j} - \mathcal{P}_{\mathbf{P}}^\perp \mathbf{P} \hat{\mathbf{h}}_{i,j}) \quad (57)$$

$$= (\mathcal{P}_{\mathbf{P}}^\perp \mathbf{y}_{i,j})^H \mathcal{P}_{\mathbf{P}}^\perp \mathbf{y}_{i,j} \quad (58)$$

$$= \mathbf{y}_{i,j}^H \mathcal{P}_{\mathbf{P}}^\perp \mathbf{y}_{i,j}, \quad (59)$$

where the projection matrix  $\mathcal{P}_{\mathbf{P}}^\perp$  in (55) is defined as  $\mathcal{P}_{\mathbf{P}}^\perp = \mathbf{I} - \mathbf{P} (\mathbf{P}^H \mathbf{P})^{-1} \mathbf{P}^H$ . The simplification in (56), (57), (58) and (59) follows using  $\mathbf{v}_{i,j} = \mathbf{y}_{i,j} - \mathbf{P} \hat{\mathbf{h}}_{i,j}$  and the properties  $(\mathcal{P}_{\mathbf{P}}^\perp)^H = \mathcal{P}_{\mathbf{P}}^\perp$ ,  $\mathcal{P}_{\mathbf{P}}^\perp \mathbf{P} = \mathbf{0}$  and  $\mathcal{P}_{\mathbf{P}}^\perp \mathcal{P}_{\mathbf{P}}^\perp = \mathcal{P}_{\mathbf{P}}^\perp$  of the projection matrix, respectively. Starting from the GLRT test statistic in (60), one can further obtain the expressions in (61) and (62). Finally, at low SNR, the non-antipodal signaling based robust GLRT (N-RGLRT) detector for cooperative spectrum sensing in the MIMO cognitive radio networks can be approximated as,

$$T_{\text{N-RGLRT}}(\mathbf{y}) = \sum_{i=1}^N a_i \sum_{j=1}^{N_f} \mathbf{y}_{i,j}^H \mathcal{P}_{\mathbf{P}} \mathbf{y}_{i,j}, \quad (63)$$

where (61) is obtained using simplification (59) in the PDFs  $p(\mathbf{y}_{i,j} | \mathbf{U}_i = \mathbf{P}; \hat{\mathbf{e}}_{i,j})$ ,  $p(\mathbf{y}_{i,j} | \mathbf{U}_i = \mathbf{0})$  corresponding to the two hypotheses. Simplification in (62) follow using the property  $\mathcal{P}_{\mathbf{P}} = \mathbf{I} - \mathcal{P}_{\mathbf{P}}^\perp$  of projection matrices and the scalar  $a_i$  in (63) is defined as  $a_i = P_{D,i} - P_{F,i}$ . The test statistic  $T_{\text{N-RGLRT-I}}(\mathbf{y})$  for identical local detection performance of the

cooperating users, i.e.,  $P_{D,i} = P_d$  and  $P_{F,i} = P_f$ ,  $\forall i$ , can be obtained from (63) as,

$$T_{\text{N-RGLRT-I}}(\mathbf{y}) = \sum_{i=1}^N \sum_{j=1}^{N_f} \mathbf{y}_{i,j}^H \mathcal{P}_{\mathbf{P}} \mathbf{y}_{i,j}. \quad (64)$$

Next, we derive the probabilities of detection and false alarm for the above robust test statistic  $T_{\text{N-RGLRT-I}}(\mathbf{y})$  detector.

**Theorem 4.** *The probabilities of detection  $P_D$  and false alarm  $P_{FA}$  for the non-antipodal signaling based robust GLRT detector  $T_{\text{N-RGLRT-I}}(\mathbf{y})$  in (64) towards cooperative spectrum sensing in MIMO cognitive radio scenarios can be given as,*

$$P_{FA} = Q_{\chi_{2RN N_f}^2} \left( \frac{2\gamma}{\sigma^2} \right), \quad (65)$$

$$P_D = Q_{\chi_{2RN N_f}^2(\theta)} \left( \frac{2\gamma}{\sigma^2} \right), \quad (66)$$

where  $\gamma$  denotes the detection threshold,  $\sigma^2$  the noise variance and  $\theta$  the non-centrality parameter defined as  $\theta \triangleq \sum_{i=1}^N \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2$ .

*Proof.* See Appendix D.  $\square$

Next, we present antipodal signaling based GLRT detector for cooperative spectrum sensing in MIMO cognitive radio networks.

### B. Antipodal signaling

Let the transmitted matrices be  $\mathbf{P}_1 = +\mathbf{P}$  and  $\mathbf{P}_0 = -\mathbf{P}$  that correspond to the local decisions regarding the presence and absence of the primary user signal. For this antipodal signaling format, the conventional GLRT approach above for the non-antipodal signaling format fails. For this scenario, the GLRT  $\frac{p(\mathbf{y}_{i,j} | \mathcal{H}_1; \hat{\mathbf{e}}_{i,j|1})}{p(\mathbf{y}_{i,j} | \mathcal{H}_0; \hat{\mathbf{e}}_{i,j|0})}$  obtained by employing the estimates of the uncertainty vectors  $\hat{\mathbf{e}}_{i,j|1}$ ,  $\hat{\mathbf{e}}_{i,j|0}$  corresponding to the hypotheses  $\mathcal{H}_1$ ,  $\mathcal{H}_0$  respectively, reduces identically to 1. In other words, the argument of the exponential term in the GLRT reduces to zero at each receive antenna for each user. This basically arises due to the fact that the standard GLRT is concerned only with the component of  $\mathbf{y}_{i,j}$  in the null space of the decision matrix. Since the decision matrices are  $\mathbf{P}$ ,  $-\mathbf{P}$  for the antipodal signaling format, their null spaces are identical, which renders the GLRT inapplicable. The reader is referred to Section II of the technical report in [37] for a detailed proof of this fact. Hence, to overcome this challenge, the regularized



$$T_{\text{A-RGLRT}}(\mathbf{y}) = \sum_{i=1}^N \ln \left[ \frac{P_{D,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1) + (1 - P_{D,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_1)}{P_{F,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_0) + (1 - P_{F,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_0)} \right] \quad (72)$$

$$= \sum_{i=1}^N \ln \left[ \frac{P_{D,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1) + (1 - P_{D,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_1)}{P_{F,i} \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_0) + (1 - P_{F,i}) \prod_{j=1}^{N_f} p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|0} | \mathbf{U}_i = -\mathbf{P}; \mathcal{H}_0)} \right] \quad (73)$$

$$= \sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \prod_{j=1}^{N_f} \frac{1}{\pi^{N_c+L} |\mathbf{R}_{z,i}|} \exp \left\{ -\mathbf{v}_{i,j|1}^H (-\mathbf{A}_{i|1} \mathbf{R}_{z,i} \mathbf{A}_{i|1}^H)^{-1} \mathbf{v}_{i,j|1} \right\}}{P_{F,i} + (1 - P_{F,i}) \prod_{j=1}^{N_f} \frac{1}{\pi^{N_c+L} |\mathbf{R}_{z,i}|} \exp \left\{ -\mathbf{v}_{i,j|0}^H (-\mathbf{A}_{i|0} \mathbf{R}_{z,i} \mathbf{A}_{i|0}^H)^{-1} \mathbf{v}_{i,j|0} \right\}} \right]. \quad (74)$$

likelihood of  $\mathbf{y}_{i,j}$  and  $\mathbf{z}_{i,j} = [\mathbf{e}_{i,j}^T \mathbf{w}_{i,j}^T]^T$  under the alternative hypothesis  $\mathcal{H}_1$  is defined as,

$$p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1) = p(\mathbf{y}_{i,j} | \mathbf{z}_{i,j|1}, \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1) p(\mathbf{z}_{i,j|1} | \mathbf{U}_i = \mathbf{P}) \quad (67)$$

$$= \frac{1}{\pi^{N_c+L} |\mathbf{R}_{z,i}|} \exp(-\mathbf{z}_{i,j|1}^H \mathbf{R}_{z,i}^{-1} \mathbf{z}_{i,j|1}), \quad (68)$$

$$\text{if } \underbrace{\mathbf{y}_{i,j} - \mathbf{P} \hat{\mathbf{h}}_{i,j|1}}_{\mathbf{v}_{i,j|1}} = \mathbf{A}_{i|1} \mathbf{z}_{i,j|1} \text{ and } 0 \text{ otherwise,}$$

where,  $\mathbf{R}_{z,i} = E \{ \mathbf{z}_{i,j} \mathbf{z}_{i,j}^H \} \in \mathbb{C}^{(N_c+L) \times (N_c+L)}$  represents the covariance matrix of the unknown vector  $\mathbf{z}_{i,j}$ , given as,

$$\mathbf{R}_{z,i} = \begin{bmatrix} \mathbf{R}_{e,i} & \mathbf{0}_{N_c \times L} \\ \mathbf{0}_{L \times N_c} & \sigma^2 \mathbf{I}_L \end{bmatrix}. \quad (69)$$

The estimate of the unknown random vector  $\hat{\mathbf{z}}_{i,j|1}$  corresponding to the alternative hypothesis  $\mathcal{H}_1$  can be obtained by maximizing the likelihood  $p(\mathbf{y}_{i,j}, \mathbf{z}_{i,j|1} | \mathbf{U}_i = \mathbf{P}; \mathcal{H}_1)$  given in (68). The likelihood maximization problem can be equivalently formulated as,

$$\begin{aligned} \min_{\mathbf{z}_{i,j|1}} \quad & \mathbf{z}_{i,j|1}^H \mathbf{R}_{z,i}^{-1} \mathbf{z}_{i,j|1} \\ \text{s.t.} \quad & \mathbf{v}_{i,j|1} = \mathbf{A}_{i|1} \mathbf{z}_{i,j|1}. \end{aligned} \quad (70)$$

It is worth noting that the above weighted norm optimization problem in (70) is convex and its solution [38], i.e the estimate of  $\mathbf{z}_{i,j|1}$  corresponding to the alternative hypothesis  $\mathcal{H}_1$ , can be readily obtained as,

$$\hat{\mathbf{z}}_{i,j|1} = \mathbf{R}_{z,i} \mathbf{A}_{i|1}^H (\mathbf{A}_{i|1} \mathbf{R}_{z,i} \mathbf{A}_{i|1}^H)^{-1} \mathbf{v}_{i,j}. \quad (71)$$

Similarly, the estimate of the unknown random vector  $\hat{\mathbf{z}}_{i,j|0}$  corresponding to the null hypothesis  $\mathcal{H}_0$  is obtained as  $\hat{\mathbf{z}}_{i,j|0} = \mathbf{R}_{z,i} \mathbf{A}_{i|0}^H (\mathbf{A}_{i|0} \mathbf{R}_{z,i} \mathbf{A}_{i|0}^H)^{-1} \mathbf{v}_{i,j}$ . Hence, using the PDFs with the obtained maximum likelihood estimates (MLEs), i.e.,  $p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|1} | \mathbf{U}_i = \mathbf{P})$ ,  $p(\mathbf{y}_{i,j}, \hat{\mathbf{z}}_{i,j|0} | \mathbf{U}_i = -\mathbf{P})$ , in (73) for the antipodal signaling case, i.e., mapping  $\mathbf{P}_1 = +\mathbf{P}$  and  $\mathbf{P}_0 = -\mathbf{P}$ , the LRT can be determined as shown in (74). Further simplifying for low SNR, the robust generalized likelihood ratio test (A-RGLRT) can be obtained as,

$$T_{\text{A-RGLRT}}(\mathbf{y}) = \sum_{i=1}^N \left[ a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{\Gamma}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}) \right]. \quad (75)$$

where  $\mathbf{\Gamma}_i = \mathbf{P} \mathbf{R}_{e,i} \mathbf{P}^H + \sigma^2 \mathbf{I}_L$  and Equation (74) is obtained using the low SNR approximation. Detailed derivation of the robust GLRT above for antipodal signaling is given in Section

III of the technical report in [37]. It can be observed that the GLRT based robust test statistic above is identical to the uncertainty statistics-based robust detector obtained in (40). Hence, the detection performance of the RGLRT is identical to that obtained in Theorem 3, as the expressions for the mean and variance derived in (45), (47) and (46), (48) remain unchanged. Further, for scenarios with unknown CSI uncertainty covariance, a simpler test which incorporates a regularization parameter  $\lambda$  is given in Section IV of the technical report in [37]. Next section presents simulation results to illustrate the spectrum sensing performance of the proposed schemes.

## VI. SIMULATION RESULTS

The detection performances of various detectors proposed in this paper are compared next. A cooperative MIMO cognitive radio scenario is considered with  $N_c=2$  transmit antennas at the secondary user and  $N_f=2$  receive antennas at the fusion center, thus leading to a  $2 \times 2$  MIMO system between the fusion center and each of the secondary users. The MIMO channels between the cooperating users and the fusion center are assumed to comprise of iid Rayleigh fading coefficients of average gain unity  $\sim \mathcal{CN}(0, 1)$ . Large scale fading is ignored similar to other works such as [21], [22]. The local performance indices for each of the secondary users are set as  $P_{D,i}=0.95$  and  $P_{F,i}=0.01$ ,  $1 \leq i \leq N$ , unless stated otherwise. The uncertainty covariance matrix  $\mathbf{R}_{e,i}$  is modeled as  $\mathbf{R}_{e,i} = \sigma_{e,i}^2 \mathcal{D}(\mathbf{b}) \in \mathbb{R}^{2 \times 2}$ ,  $1 \leq i \leq N$ , where the constant  $\sigma_{e,i}^2$  takes values  $\sigma_{e,i}^2 \in \{0.33, 0.83, 1\}$  and  $\mathcal{D}(\mathbf{b})$  denotes a diagonal matrix with elements of vector  $\mathbf{b}$  along its principal diagonal. For simulations, the transmitted matrix  $\mathbf{P}$ , of dimension  $L \times 2$  is chosen as an orthogonal matrix, i.e.,  $\mathbf{P}^H \mathbf{P} = \mathbf{I}_2$ .

Fig. 2 to Fig. 5c present plots of the probability of detection ( $P_D$ ) versus probability of false alarm ( $P_{FA}$ ), i.e., receiver operating characteristic (ROC) of the proposed detection schemes for different scenarios. Fig. 2 presents a comparison of the detection performance for different values of  $\text{SNR} \in \{-10, -2, 15\}$  dB at the fusion center. It can be observed from both Fig. 2a and Fig. 2b that the proposed robust detector  $T_R(\mathbf{y})$  has an improved detection performance in comparison to the uncertainty agnostic  $T_{UA}(\mathbf{y})$  detector for all SNRs. Moreover, the performance gap between the robust and uncertainty agnostic detectors increases with SNR. Also, naturally, the detection performance of the presented detectors improves with an increase in the number of decision vectors  $L$ .

Simulation results in Fig. 3a compare three sets of plots for the robust detector  $T_R(\mathbf{y})$ , uncertainty agnostic

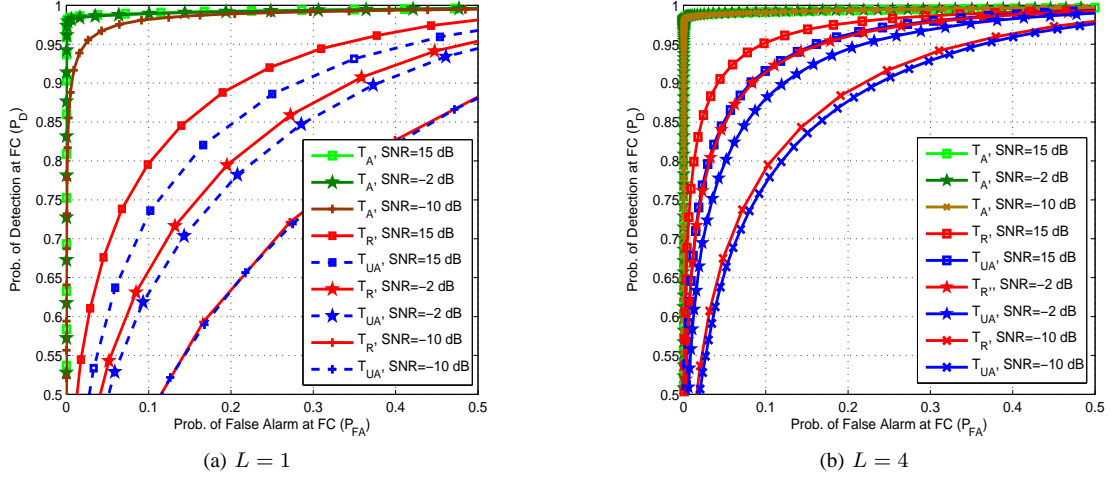


Fig. 2. ROC curves for the system with  $N = 2$  secondary users,  $N_c = 2$  transmit antennas,  $N_f = 2$  antennas at the fusion center, under antipodal signaling.

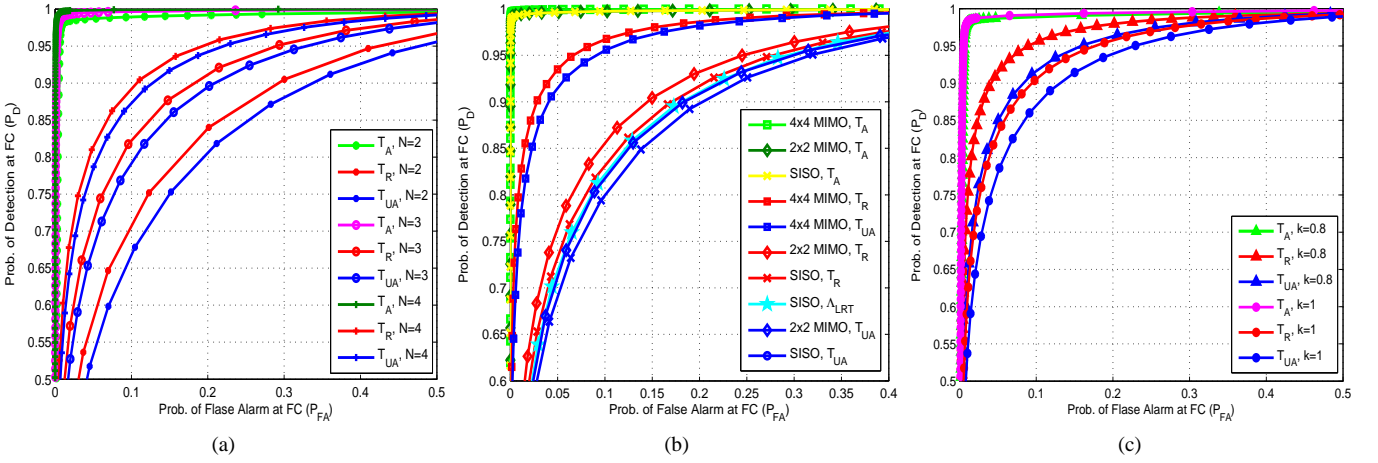


Fig. 3. ROC plots for (a)  $2 \times 2$ -MIMO network with  $N \in \{2, 3, 4\}$  users,  $N_c = 2$  transmit antennas,  $N_f = 2$ ,  $L = 1$  at  $\text{SNR}=1\text{dB}$ . (b) SISO,  $2 \times 2$ -MIMO,  $4 \times 4$ -MIMO,  $N = 3$  users,  $L = 1$  at  $\text{SNR}=1\text{dB}$ . (c)  $N = 2$  users for uncertainty levels  $k \in \{1, 0.8\}$ ,  $N_c = 2$  transmit antennas,  $N_f = 2$ ,  $L = 2$ ,  $\text{SNR}=1\text{dB}$ .

detector  $T_{UA}(\mathbf{y})$  and the CSI aware detector  $T_A(\mathbf{y})$  for increasing number of cooperating secondary users  $N \in \{2, 3, 4\}$ . Each set of plots considers  $N$  cooperating secondary users under CSI uncertainty level  $\sigma_{e,i}^2$  set as  $\sigma_{e,i}^2 \in \{\sigma_{e,1}^2, \dots, \sigma_{e,i}^2, \dots, \sigma_{e,N}^2\}$ ,  $1 \leq i \leq N$ , evenly spaced between  $\sigma_{e,1}^2 = 1$  and  $\sigma_{e,N}^2 = 0.33$ . It is evident from Fig. 3a that the performance of all the schemes improves with an increase in the number of cooperating secondary users and worsens with increasing uncertainty. Further, as expected, perfect CSI-based  $T_A(\mathbf{y})$  and uncertainty agnostic  $T_{UA}(\mathbf{y})$  have the best and worst performances, while the ROC of the robust test statistic  $T_R(\mathbf{y})$  lies between those of the above two. Fig. 3b compares the detection performance of proposed detectors for varying values of receive and transmit antennas along with the detector  $\Lambda_{LRT}$  presented in [39] for SISO scenarios. For the SISO scenario, the proposed robust detector  $T_R(\mathbf{y})$  outperforms the SISO based LRT detector  $\Lambda_{LRT}$  in [39]. Further, it can be observed that as the number of receive and transmit antennas increases, the detection performance of the proposed robust detection schemes improves significantly.

Fig. 3c presents the trend in the performance of the proposed detectors for a varying CSI uncertainty level with  $N = 2$  secondary users and  $L = 2$  antipodal decision vectors at  $\text{SNR} = 1$  dB. The uncertainty level  $\sigma_{e,i}^2$  for  $N = 2$  secondary users is set as  $k(\sigma_{e,1}^2, \sigma_{e,2}^2)$  where  $k \in \{1, 0.8\}$  with  $\sigma_{e,1}^2 = 1$  and  $\sigma_{e,2}^2 = 0.33$ . It is observed in Fig. 3c that the performance of the proposed detectors improves with decrease in the CSI uncertainty level. Further, it can be observed that as the CSI uncertainty level increases, the performance gap between the robust detector and, the uncertainty agnostic detector progressively increases.

Plots in Fig. 4a present the probability of detection vs. the probability of false alarm at the fusion center for the CSI aware ( $T_A(\mathbf{y})$ ), uncertainty statistics-based robust ( $T_R(\mathbf{y})$ ) and the uncertainty agnostic ( $T_{UA}(\mathbf{y})$ ) detectors obtained in (15), (41) and (125) respectively for  $L = 1, 2$ . The CSI aware scheme in this case serves as an upper bound to characterize the detection performance of the other proposed robust and non-robust schemes. It can be observed that the proposed uncertainty aware robust detector and the robust GLRT detector have

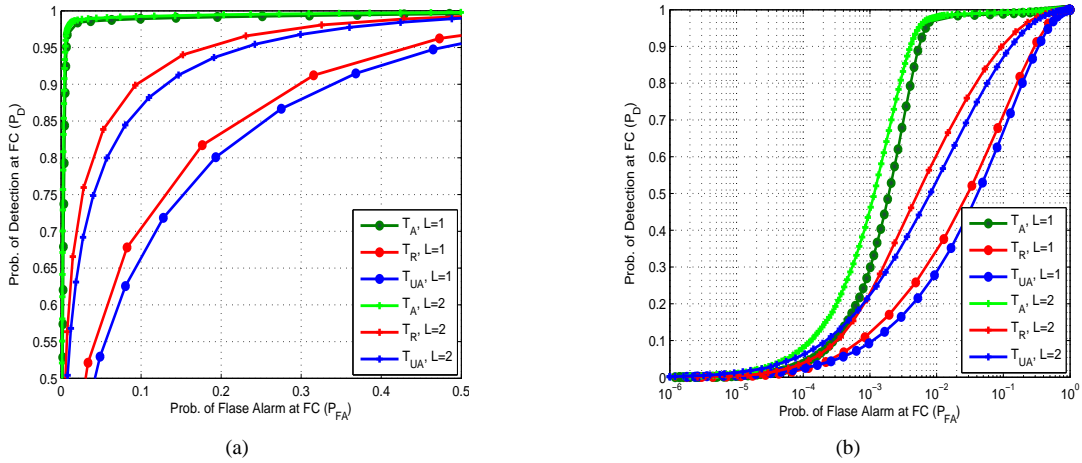


Fig. 4. ROC plots for a  $2 \times 2$ -MIMO network with  $N = 2$  secondary users,  $(\sigma_{e,1}^2, \sigma_{e,2}^2) = (1, 0.33)$ ,  $N_c = 2$ ,  $N_f = 2$ ,  $L \in \{1, 2\}$ , at SNR = 1dB and antipodal signaling. (a)  $P_{FA}$  in linear scale, (b)  $P_{FA}$  in logarithmic scale.

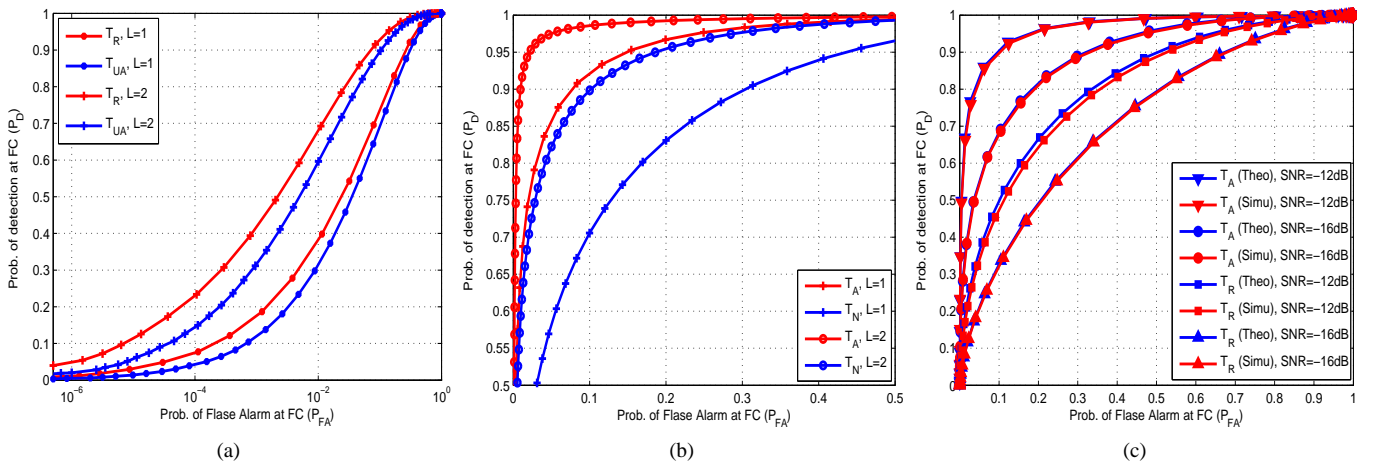


Fig. 5. ROC plots for a MIMO network with  $N = 2$  secondary users,  $N_c = 2$  transmit antennas,  $N_f = 2$ , (a) under antipodal signaling with  $(\sigma_{e,1}^2, \sigma_{e,2}^2) = (1, 0.33)$ , and  $(P_{D,i}, P_{F,i}) = (1, 0)$ ,  $L \in \{1, 2\}$  at SNR = 1 dB, (b) antipodal and non-antipodal signaling  $L \in \{1, 2\}$ , SNR = -5 dB, (c) antipodal signaling with  $(\sigma_{e,1}^2, \sigma_{e,2}^2) = (1, 0.33)$  and  $L = 1$ .

a significantly improved performance in comparison to the uncertainty agnostic detector in (125). Fig. 4b presents the ROC plots in logarithmic scale to present finer details of the performance at a higher resolution. It is seen from Fig. 4b that there is a slight performance degradation in the robust detector for  $L = 2$  over a small range of very low probabilities of false alarm, i.e.,  $(P_{FA} < 10^{-3})$ , at the fusion center. This arises basically due to the low SNR approximations made for the exponential and log functions while deriving the detectors. To further investigate this, Fig. 5a presents the ROC performance for a scenario similar to the one considered in Fig. 5 with ideal local detection performance, i.e.,  $P_{D,i} = 1$  and  $P_{F,i} = 0, \forall i$ . It can be observed, that for such a scenario the proposed robust detector always yields an improved detection performance with respect to the uncertainty agnostic detector. This is due to the fact that the test in (14) for perfect local detection at the constituent sensing nodes in the cooperative sensing network leads to in the test statistic in (15) for all SNRs without the need for the low SNR approximation.

The detection performance of the CSI aware detector employing antipodal signaling, i.e.,  $\mathbf{P}_1 = \mathbf{P}$ ,  $\mathbf{P}_0 = -\mathbf{P}$ , and non-antipodal signaling, i.e.,  $\mathbf{P}_1 = \sqrt{2}\mathbf{P}$ ,  $\mathbf{P}_0 = \mathbf{0}_{L \times N_c}$ , schemes are presented in Fig. 5b. Results shows that the antipodal signaling scheme has a better performance, thus making it well suited for cooperative sensing scenarios. For the same power level, the deflection coefficient [30] for antipodal signaling is greater than the non-antipodal signaling, which leads to the improved sensing performance for the former signaling scheme. Finally, Fig. 5c presents a comparison of the simulated ROC plots with their respective analytical counterparts obtained using the expressions for  $P_D$  and  $P_{FA}$  in Theorem 1 and, Theorem 2 of Section III and Theorem 3 of Section IV for  $L=1$  and  $\text{SNR} \in \{-12, -16\}$  dB for the perfect CSI scenario in (15), (23) and the robust detector in (41). It is observed that the simulation and analytical performance curves coincide, thus validating the analytical expressions derived to characterize the various probabilities of detection and false alarm. Also the performance of the proposed robust GLRT scheme has

not been presented in this figure, as it is evident from (75) that the performance of the robust GLRT is identical to that of the uncertainty statistics-based robust detector in (41).

## VII. CONCLUSION

This paper addressed the problem of cooperative spectrum sensing in multiuser cognitive radio networks with multiple transmit antennas at each secondary user and multiple receive antennas at the fusion center. Cooperative detection rules based on local decisions transmitted by the cooperating secondary users have been derived initially for the scenario with perfect CSI at the fusion center. Subsequently, robust detectors have also been presented for a more practical scenario under CSI uncertainty in cooperative MIMO cognitive radio networks. Closed form expressions have been obtained for the probabilities of detection ( $P_D$ ) and false alarm ( $P_{FA}$ ) to analytically characterize the performance of the proposed detectors. Simulation results demonstrate that the proposed uncertainty aware robust detectors have an improved performance in comparison to the conventional uncertainty agnostic detector and also validate the analytical results. The key insights from this study are as follows. Firstly, it is evident that multi-user MIMO cooperative spectrum sensing yields a significant performance improvement over single-user non-cooperative, single antenna schemes, especially in the low SNR regime. Further, the derived fusion rules in the low SNR regime for scenarios with and without CSI uncertainty have a linear complexity, which renders them suitable for practical systems. Finally, the fusion rules with CSI uncertainty lead to a significant improvement in sensing performance over uncertainty agnostic schemes.

This work focused on obtaining robust decision rules at the fusion center based on local decisions received from the cooperating secondary users considering CSI uncertainty in cooperative MIMO Cognitive Radio Networks. An interesting extension of the present work would be to incorporate censoring of local decisions prior to transmission and subsequent combining at the fusion center.

### APPENDIX A

#### PROOF OF THEOREM 1

The mean of the test statistic  $T_A(\mathbf{y})$  in (15) under the alternative hypothesis  $\mathcal{H}_1$  is defined as,

$$\mu_{T_A|\mathcal{H}_1} = E\{T_A(\mathbf{y})|\mathcal{H}_1\} = \sum_{i=1}^N E\{\mathbb{T}(\mathbf{y}_i)|\mathcal{H}_1\}, \quad (76)$$

where the mean of each term  $\mathbb{T}(\mathbf{y}_i)$  corresponding to the alternative hypothesis is obtained as,

$$E\{\mathbb{T}(\mathbf{y}_i)|\mathcal{H}_1\} = a_i \sum_{j=1}^{N_f} \Re(E\{\mathbf{y}_{i,j}|\mathcal{H}_1\}^H \mathbf{P} \mathbf{h}_{i,j}) \quad (77)$$

$$= a_i \sum_{j=1}^{N_f} \Re(\mathbf{h}_{i,j}^H E\{\mathbf{U}_i|\mathcal{H}_1\}^H \mathbf{P} \mathbf{h}_{i,j}) \quad (78)$$

$$= a_i \sum_{j=1}^{N_f} \Re\left(\mathbf{h}_{i,j}^H \underbrace{(\mathbf{P} \Pr(\mathbf{U}_i = \mathbf{P}) + (-\mathbf{P}) \Pr(\mathbf{U}_i = -\mathbf{P}))}_{E\{\mathbf{U}_i|\mathcal{H}_1\}}^H \mathbf{P} \mathbf{h}_{i,j}\right) \quad (79)$$

Since  $\Pr(\mathbf{U}_i = \mathbf{P}|\mathcal{H}_1) = P_{D,i}$  and  $\Pr(\mathbf{U}_i = -\mathbf{P}|\mathcal{H}_1) = 1 - P_{D,i}$ ,  $E\{\mathbf{U}_i|\mathcal{H}_1\} = P_{D,i} \mathbf{P} + (1 - P_{D,i})(-\mathbf{P}) = (2P_{D,i} - 1)\mathbf{P}$ . Hence, the expression above can be simplified as,

$$E\{\mathbb{T}(\mathbf{y}_i)|\mathcal{H}_1\} = a_i \sum_{j=1}^{N_f} \Re(\mathbf{h}_{i,j}^H (2P_{D,i} - 1) \mathbf{P}^H \mathbf{P} \mathbf{h}_{i,j}) \quad (80)$$

$$= a_i (2P_{D,i} - 1) \sum_{j=1}^{N_f} \Re(\mathbf{h}_{i,j}^H \mathbf{P}^H \mathbf{P} \mathbf{h}_{i,j}) \quad (81)$$

$$= a_i b_i \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2, \quad (82)$$

where  $b_i = 2P_{D,i} - 1$ . From (76) and (82), the mean of the test statistic in (15) corresponding to the hypothesis  $\mathcal{H}_1$  can be obtained as,

$$\mu_{T_A|\mathcal{H}_1} = \sum_{i=1}^N \left[ a_i b_i \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right]. \quad (83)$$

Similarly the mean  $\mu_{T_A|\mathcal{H}_0}$  corresponding to the null hypothesis  $\mathcal{H}_0$  can be obtained as,

$$\mu_{T_A|\mathcal{H}_0} = \sum_{i=1}^N \left[ a_i c_i \sum_{j=1}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right], \quad (84)$$

where  $c_i = 2(P_{F,i} - 1)$ . The variance of the test statistic  $T_A(\mathbf{y})$  under the alternative hypothesis  $\mathcal{H}_1$  is,

$$\sigma_{T_A|\mathcal{H}_1}^2 = \sum_{i=1}^N \sigma_{\mathbb{T}_i|\mathcal{H}_1}^2 = \sum_{i=1}^N [E\{\mathbb{T}^2(\mathbf{y}_i)|\mathcal{H}_1\} - E\{\mathbb{T}(\mathbf{y}_i)|\mathcal{H}_1\}^2]. \quad (85)$$

The first term in (85) can be simplified as,

$$E\{\mathbb{T}^2(\mathbf{y}_i)|\mathcal{H}_1\} = E\left\{ \left[ a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}) \right]^2 | \mathcal{H}_1 \right\} \quad (86)$$

$$= a_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} E\{\Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}) | \mathcal{H}_1\} E\{\Re(\mathbf{y}_{i,k}^H \mathbf{P} \mathbf{h}_{i,k}) | \mathcal{H}_1\} \\ + a_i^2 \sum_{j=1}^{N_f} E\left\{ (\Re(\mathbf{y}_{i,j}^H \mathbf{P} \mathbf{h}_{i,j}))^2 | \mathcal{H}_1 \right\} \quad (87)$$

$$= a_i^2 b_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \|\mathbf{P} \mathbf{h}_{i,k}\|^2 \\ + a_i^2 \sum_{j=1}^{N_f} \left[ \|\mathbf{P} \mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P} \mathbf{h}_{i,j}\|^2 \right]. \quad (88)$$

Equation (87) follows from the independence of the signals received at the different antennas. Substituting the expressions

for  $E\{\mathbb{T}(\mathbf{y}_i)|\mathcal{H}_1\}$ ,  $E\{\mathbb{T}^2(\mathbf{y}_i)|\mathcal{H}_1\}$  from (82), (88) in (85) yields,

$$\begin{aligned} \sigma_{\mathbb{T}_i|\mathcal{H}_1}^2 &= \sum_{i=1}^N a_i^2 \left[ b_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \|\mathbf{P}\mathbf{h}_{i,k}\|^2 \right. \\ &\quad \left. + \sum_{j=1}^{N_f} \left[ \|\mathbf{P}\mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right] - \left[ b_i \sum_{j=1}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right]^2 \right]. \end{aligned} \quad (89)$$

Employing the expression for  $\sigma_{\mathbb{T}_i|\mathcal{H}_1}^2$  above in (85), one obtains the variance  $\sigma_{T_A|\mathcal{H}_1}^2$  as given in (20). The variance  $\sigma_{T_A|\mathcal{H}_0}^2$  of the test statistic  $T_A(\mathbf{y})$  in (15) corresponding to the null hypothesis  $\mathcal{H}_0$ , can be obtained similarly by setting,

$$\begin{aligned} \sigma_{T_A|\mathcal{H}_0}^2 &= \sum_{i=1}^N a_i^2 \left[ c_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \|\mathbf{P}\mathbf{h}_{i,k}\|^2 \right. \\ &\quad \left. + \sum_{j=1}^{N_f} \left[ \|\mathbf{P}\mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right] - \left[ c_i \sum_{j=1}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right]^2 \right], \end{aligned} \quad (90)$$

where  $c_i = (2P_{F,i} - 1)$ . Finally, the  $P_{FA}$  and  $P_D$  expressions at the fusion center for cooperative spectrum sensing under antipodal signaling can be obtained as,

$$\begin{aligned} P_{FA} &= \Pr(T_A > \gamma | \mathcal{H}_0) = Q\left(\frac{\gamma - \mu_{T_A|\mathcal{H}_0}}{\sigma_{T_A|\mathcal{H}_0}}\right), \\ P_D &= \Pr(T_A > \gamma | \mathcal{H}_1) = Q\left(\frac{\gamma - \mu_{T_A|\mathcal{H}_1}}{\sigma_{T_A|\mathcal{H}_1}}\right). \end{aligned}$$

## APPENDIX B PROOF OF THEOREM 2

The mean of the test statistic  $T_N(\mathbf{y})$  in (23) under the alternative hypothesis  $\mathcal{H}_1$  is obtained as,

$$\mu_{T_N|\mathcal{H}_1} = \sum_{i=1}^N E\{\mathcal{T}(\mathbf{y}_i)|\mathcal{H}_1\} \quad (91)$$

$$= \sum_{i=1}^N \left[ a_i \sum_{j=1}^{N_f} \Re\{E\{\mathbf{y}_{i,j}|\mathcal{H}_1\}^H \mathbf{P}\mathbf{h}_{i,j}\} \right] \quad (92)$$

$$= \sum_{i=1}^N \left[ a_i P_{D,i} \sum_{j=1}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right]. \quad (93)$$

Similarly the mean  $\mu_{T_N|\mathcal{H}_0}$  under the null hypothesis  $\mathcal{H}_0$  can be obtained as,

$$\mu_{T_N|\mathcal{H}_0} = \sum_{i=1}^N \left[ a_i P_{F,i} \sum_{j=1}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right]. \quad (94)$$

The variance of  $T_N(\mathbf{y})$  under  $\mathcal{H}_1$  can be expressed as,

$$\sigma_{T_N|\mathcal{H}_1}^2 = \sum_{i=1}^N \sigma_{\mathbb{T}_i|\mathcal{H}_1}^2 \quad (95)$$

$$= \sum_{i=1}^N \left( E\{\mathcal{T}^2(\mathbf{y}_i)|\mathcal{H}_1\} - E\{\mathcal{T}(\mathbf{y}_i)|\mathcal{H}_1\}^2 \right). \quad (96)$$

The first term in (96) above can be evaluated as,

$$E\{\mathcal{T}^2(\mathbf{y}_i)|\mathcal{H}_1\} = E\left\{ \left[ a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \mathbf{P}\mathbf{h}_{i,j}) \right]^2 | \mathcal{H}_1 \right\} \quad (97)$$

$$\begin{aligned} &= a_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} E\{\Re(\mathbf{y}_{i,j}^H \mathbf{P}\mathbf{h}_{i,j}) | \mathcal{H}_1\} E\{\Re(\mathbf{y}_{i,k}^H \mathbf{P}\mathbf{h}_{i,k}) | \mathcal{H}_1\} \\ &\quad + a_i^2 \sum_{j=1}^{N_f} E\{(\Re(\mathbf{y}_{i,j}^H \mathbf{P}\mathbf{h}_{i,j}))^2 | \mathcal{H}_1\} \end{aligned} \quad (98)$$

$$\begin{aligned} &= a_i^2 P_{D,i}^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \|\mathbf{P}\mathbf{h}_{i,k}\|^2 \\ &\quad + a_i^2 \sum_{j=1}^{N_f} \left[ P_{D,i}^2 \|\mathbf{P}\mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right]. \end{aligned} \quad (99)$$

Substituting the expression for  $E\{\mathcal{T}(\mathbf{y}_i)|\mathcal{H}_1\}$  from (92) and  $E\{\mathcal{T}^2(\mathbf{y}_i)|\mathcal{H}_1\}$  from (99) in (96) the variance  $\sigma_{\mathbb{T}_i|\mathcal{H}_1}^2$  corresponding to the  $i$ th term in (95) can be obtained as,

$$\begin{aligned} \sigma_{\mathbb{T}_i|\mathcal{H}_1}^2 &= \sum_{i=1}^N a_i^2 \left[ P_{D,i}^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \|\mathbf{P}\mathbf{h}_{i,k}\|^2 \right. \\ &\quad \left. + \sum_{j=1}^{N_f} \left[ P_{D,i}^2 \|\mathbf{P}\mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right] - \left[ P_{D,i} \sum_{j=1}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right]^2 \right]. \end{aligned} \quad (100)$$

Using the expression for  $\sigma_{\mathbb{T}_i|\mathcal{H}_1}^2$  in (95) yields the expression for variance  $\sigma_{T_N|\mathcal{H}_1}^2$  under the alternative hypothesis  $\mathcal{H}_1$  in (28). Similarly, the variance  $\sigma_{T_N|\mathcal{H}_0}^2$  under the null hypothesis  $\mathcal{H}_0$  can be obtained as,

$$\begin{aligned} \sigma_{T_N|\mathcal{H}_0}^2 &= \sum_{i=1}^N a_i^2 \left[ P_{F,i}^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \|\mathbf{P}\mathbf{h}_{i,k}\|^2 \right. \\ &\quad \left. + \sum_{j=1}^{N_f} \left[ P_{F,i}^2 \|\mathbf{P}\mathbf{h}_{i,j}\|^4 + \frac{\sigma^2}{2} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right] - \left[ P_{F,i} \sum_{j=1}^{N_f} \|\mathbf{P}\mathbf{h}_{i,j}\|^2 \right]^2 \right]. \end{aligned} \quad (101)$$

The expressions of  $\mu_{T_N|\mathcal{H}_0}$  and  $\sigma_{T_N|\mathcal{H}_0}^2$  under the null hypothesis can be similarly obtained by replacing  $P_{D,i}$  with  $P_{F,i}$  in  $\mu_{T_N|\mathcal{H}_1}$  and  $\sigma_{T_N|\mathcal{H}_1}^2$  respectively. With means and variances obtained as described above, the probability of false alarm  $P_{FA}$  and the probability of detection  $P_D$  at the fusion center can be obtained as,

$$\begin{aligned} P_{FA} &= \Pr(T_N > \gamma | \mathcal{H}_0) = Q\left(\frac{\gamma - \mu_{T_N|\mathcal{H}_0}}{\sigma_{T_N|\mathcal{H}_0}}\right), \\ P_D &= \Pr(T_N > \gamma | \mathcal{H}_1) = Q\left(\frac{\gamma - \mu_{T_N|\mathcal{H}_1}}{\sigma_{T_N|\mathcal{H}_1}}\right). \end{aligned}$$

APPENDIX C  
PROOF OF THEOREM 3

The mean of the test statistic  $T_R(\mathbf{y})$  in (41) under the alternative hypothesis  $\mathcal{H}_1$  is given as,

$$\begin{aligned} \mu_{T_R|\mathcal{H}_1} &= \sum_{i=1}^N E\{\mathsf{T}(\mathbf{y}_i)|\mathcal{H}_1\} \\ &= \sum_{i=1}^N \left[ a_i \sum_{j=1}^{N_f} \Re\{E\{\mathbf{y}_{i,j}|\mathcal{H}_1\}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\} \right] \end{aligned} \quad (102)$$

$$= \sum_{i=1}^N \left[ a_i b_i \sum_{j=1}^{N_f} \Re\{\hat{\mathbf{h}}_{i,j}^H \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\} \right] \quad (103)$$

$$= \sum_{i=1}^N \left[ a_i b_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right], \quad (104)$$

where  $\Gamma_i = \mathbf{Q}_i \mathbf{Q}_i^H$ . The variance of the detector  $T_R(\mathbf{y})$  under the alternative hypothesis  $\mathcal{H}_1$  is defined as,

$$\begin{aligned} \sigma_{T_R|\mathcal{H}_1}^2 &= \sum_{i=1}^N \sigma_{\mathsf{T}_i|\mathcal{H}_1}^2 = \sum_{i=1}^N \underbrace{[E\{\mathsf{T}^2(\mathbf{y}_i)|\mathcal{H}_1\} - E\{\mathsf{T}(\mathbf{y}_i)|\mathcal{H}_1\}^2]}_{\sigma_{\mathsf{T}_i|\mathcal{H}_1}^2}, \end{aligned} \quad (105)$$

where, the first term in  $\sigma_{\mathsf{T}_i|\mathcal{H}_1}^2$  can be simplified as,

$$\begin{aligned} E\{\mathsf{T}^2(\mathbf{y}_i)|\mathcal{H}_1\} &= a_i^2 \sum_{j=1}^{N_f} \sum_{k=1}^{N_f} E\{\Re\{\mathbf{y}_{i,j}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\} | \mathcal{H}_1\} E\{\Re\{\mathbf{y}_{i,k}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,k}\} | \mathcal{H}_1\} \\ &\quad + a_i^2 \sum_{j=1}^{N_f} E\{\Re\{\mathbf{y}_{i,j}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\}^2 | \mathcal{H}_1\} \end{aligned} \quad (106)$$

$$\begin{aligned} &= a_i^2 \sum_{j=1}^{N_f} \sum_{k=1}^{N_f} \Re\{E\{\mathbf{y}_{i,j}^H | \mathcal{H}_1\} \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\} \Re\{E\{\mathbf{y}_{i,k}^H | \mathcal{H}_1\} \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,k}\} \\ &\quad + a_i^2 \sum_{j=1}^{N_f} E\left\{ \left[ \Re\{\hat{\mathbf{h}}_{i,j}^H \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\} + \Re\{\mathbf{u}_{i,j}^H \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\} \right. \right. \\ &\quad \left. \left. + \Re\{\mathbf{w}_{i,j}^H \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\} \right]^2 | \mathcal{H}_1 \right\} \end{aligned} \quad (107)$$

$$\begin{aligned} &= a_i^2 \left[ b_i^2 \sum_{j=1}^{N_f} \sum_{k=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,k}\|^2 + \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right. \\ &\quad \left. + \frac{1}{2} \|\mathbf{C}_i^{-1} \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 + \frac{\sigma^2}{2} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right], \end{aligned} \quad (108)$$

with  $\mathbf{R}_{e,i} = \mathbf{C}_i \mathbf{C}_i^H$ . Substituting the expression for  $\mu_{T_R|\mathcal{H}_1}$  from (104) and  $E\{\mathsf{T}^2(\mathbf{y}_i)|\mathcal{H}_1\}$  from (108) in (105) yields the

expression for  $\sigma_{T_R|\mathcal{H}_1}^2$  as,

$$\begin{aligned} \sigma_{T_R|\mathcal{H}_1}^2 &= \sum_{i=1}^N a_i^2 \left[ b_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,k}\|^2 \right. \\ &\quad \left. + \sum_{j=1}^{N_f} \left[ \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^4 + \frac{1}{2} \|\mathbf{C}_i^{-1} \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right. \right. \\ &\quad \left. \left. + \frac{\sigma^2}{2} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right] - \left[ b_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right]^2 \right]. \end{aligned} \quad (109)$$

Similarly, the mean  $\mu_{T_R|\mathcal{H}_0}$  and the variance  $\sigma_{T_R|\mathcal{H}_0}^2$  corresponding to the null hypothesis  $\mathcal{H}_0$  can be obtained as,

$$\mu_{T_R|\mathcal{H}_0} = \sum_{i=1}^N \left[ a_i c_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right], \quad (110)$$

$$\begin{aligned} \sigma_{T_R|\mathcal{H}_0}^2 &= \sum_{i=1}^N a_i^2 \left[ c_i^2 \sum_{j=1}^{N_f} \sum_{\substack{k=1 \\ k \neq j}}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,k}\|^2 \right. \\ &\quad \left. + \sum_{j=1}^{N_f} \left[ \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^4 + \frac{1}{2} \|\mathbf{C}_i^{-1} \mathbf{P}^H \Gamma_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right. \right. \\ &\quad \left. \left. + \frac{\sigma^2}{2} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right] - \left[ c_i \sum_{j=1}^{N_f} \|\mathbf{Q}_i^{-1} \mathbf{P} \hat{\mathbf{h}}_{i,j}\|^2 \right]^2 \right], \end{aligned} \quad (111)$$

where,  $c_i = (2P_{F,i} - 1)$ . Hence, the probability of false alarm  $P_{FA}$  and the probability of detection  $P_D$  at the fusion center can be calculated as,

$$\begin{aligned} P_{FA} &= \Pr(T_R > \gamma | \mathcal{H}_0) = Q\left(\frac{\gamma - \mu_{T_R|\mathcal{H}_0}}{\sigma_{T_R|\mathcal{H}_0}}\right), \\ P_D &= \Pr(T_R > \gamma | \mathcal{H}_1) = Q\left(\frac{\gamma - \mu_{T_R|\mathcal{H}_1}}{\sigma_{T_R|\mathcal{H}_1}}\right). \end{aligned}$$

APPENDIX D  
PROOF OF THEOREM 4

The PDFs of the received signal  $\mathbf{y}_{i,j}$  in (32) corresponding to the two hypotheses for the non-antipodal signals defined in this section can be obtained as,

$$\mathcal{H}_0 : \mathbf{y}_{i,j} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_L) \quad (112)$$

$$\mathcal{H}_1 : \mathbf{y}_{i,j} \sim \mathcal{CN}(\mathbf{P} \hat{\mathbf{h}}_{i,j}, \sigma^2 \mathbf{I}_L). \quad (113)$$

Let the singular value decomposition (SVD) of the projection matrix  $\mathcal{P} = \mathbf{V} \Sigma \mathbf{V}^H$ , where the singular value matrix  $\Sigma$  is a diagonal matrix with singular values  $\nu_i$  on its principal diagonal. The first  $R$  non-zero singular values in  $\Sigma$  are  $\nu_i = 1, 1 \leq i \leq R$  where  $R = \min\{L, N_e\}$ . Therefore, the test statistic in (64) can be further simplified as,

$$T_{\text{N-RGLRT-1}}(\mathbf{y}) = \sum_{i=1}^N \sum_{j=1}^{N_f} \mathbf{y}_{i,j}^H (\mathbf{V} \Sigma \mathbf{V}^H) \mathbf{y}_{i,j} \quad (114)$$

$$= \frac{\sigma^2}{2} \sum_{i=1}^N \sum_{j=1}^{N_f} \tilde{\mathbf{y}}_{i,j}^H \Sigma \tilde{\mathbf{y}}_{i,j} = \frac{\sigma^2}{2} \sum_{i=1}^N \sum_{j=1}^{N_f} \sum_{l=1}^R |\tilde{y}_{i,j}(l)|^2, \quad (115)$$

where  $\tilde{\mathbf{y}}_{i,j} = 2\sigma^{-2} \mathbf{V}^H \mathbf{y}_{i,j} \in \mathbb{C}^{L \times 1}$  with  $\mathbf{y}_{i,j}$  defined as  $\mathbf{y}_{i,j} = [y_{i,j}^T(1), \dots, y_{i,j}^T(L)]^T$ . Corresponding to the two hypotheses, each component  $\tilde{y}_{i,j}(l)$  of  $\tilde{\mathbf{y}}_{i,j}$  follows a complex normal distribution, i.e.,  $\mathcal{H}_0 : \tilde{y}_{i,j}(l) \sim \mathcal{CN}(0, 1)$  and

$\mathcal{H}_1 : \tilde{y}_{i,j}(l) \sim \mathcal{CN}(\mathbf{p}^H(l)\hat{\mathbf{h}}_{i,j}, 1)$ , respectively. Therefore, the PDFs of the test statistic  $T_{\text{N-RGLRT-I}}(\mathbf{y})$  in (115) under both the hypotheses can be derived as,

$$\mathcal{H}_0 : \frac{2}{\sigma^2} T_{\text{N-RGLRT-I}}(\mathbf{y}) \sim \chi_{2RNN_f}^2 \quad (116)$$

$$\mathcal{H}_1 : \frac{2}{\sigma^2} T_{\text{N-RGLRT-I}}(\mathbf{y}) \sim \chi_{2RNN_f}^2(\theta), \quad (117)$$

where  $\chi_{2RNN_f}^2$  and  $\chi_{2RNN_f}^2(\theta)$  denote the central chi-squared distribution and non-central chi-squared distribution with  $2RNN_f$  degrees of freedom respectively and the non-centrality parameter  $\theta = \sum_{i=1}^N \sum_{j=1}^{N_f} \hat{\mathbf{h}}_{i,j}^H \mathbf{P}^H \mathbf{P} \hat{\mathbf{h}}_{i,j} = \sum_{i=1}^N \sum_{j=1}^{N_f} \|\hat{\mathbf{P}}\mathbf{h}_{i,j}\|^2$ . The probability of false alarm  $P_{FA}$  for detection based on the test statistic in (64) can be obtained as,

$$P_{FA} = Pr\{T_{\text{N-RGLRT-I}} > \gamma; \mathcal{H}_0\} \quad (118)$$

$$= Pr\left\{\frac{1}{\sigma^2/2} T_{\text{N-RGLRT-I}} > \frac{\gamma}{\sigma^2/2}; \mathcal{H}_0\right\} \quad (119)$$

$$= Pr\left\{\chi_{2RNN_f}^2 > \frac{2\gamma}{\sigma^2}\right\} = Q_{\chi_{2RNN_f}^2}\left(\frac{2\gamma}{\sigma^2}\right), \quad (120)$$

where (120) follows using the distribution obtained in (116) corresponding to the null hypothesis and the function  $Q_{\chi_{2RNN_f}^2}(\cdot)$  denotes the tail probability of the chi-squared distribution with  $2RNN_f$  degrees of freedom. The probability of detection  $P_D$  for detection based on  $T_{\text{E-RGLRT}}(\mathbf{y})$  in (64) is obtained as,

$$P_D = Pr\{T_{\text{N-RGLRT-I}} > \gamma; \mathcal{H}_1\} \quad (121)$$

$$= Pr\left\{\frac{1}{\sigma^2/2} T_{\text{N-RGLRT-I}} > \frac{\gamma}{\sigma^2/2}; \mathcal{H}_1\right\} \quad (122)$$

$$= Pr\left\{\chi_{2RNN_f}^2(\theta) > \frac{2\gamma}{\sigma^2}\right\} = Q_{\chi_{2RNN_f}^2(\theta)}\left(\frac{2\gamma}{\sigma^2}\right), \quad (123)$$

where (123) follows from the non-central chi-squared distribution, obtained in (117), of the test statistic  $T_{\text{N-RGLRT}}(\mathbf{y})$ . The function  $Q_{\chi_{2RNN_f}^2(\theta)}(\cdot)$  denotes the tail probability of the non-central chi-squared distribution with  $2RNN_f$  degrees of freedom and non-centrality parameter  $\theta = \sum_{i=1}^N \sum_{j=1}^{N_f} \|\hat{\mathbf{P}}\mathbf{h}_{i,j}\|^2$ .

## APPENDIX E

### UNCERTAINTY AGNOSTIC (UA) DETECTOR

Consider an uncertainty agnostic detector at the fusion center which employs only the nominal channel estimates  $\hat{\mathbf{H}}_i$  ignoring the uncertainty for the system model described at the fusion center in (30). In such a scenario, the antipodal signaling based UA test statistic at the fusion center, based only on the nominal CSI estimate  $\hat{\mathbf{H}}_i$ , be given from (39) as,

$$T_{\text{UA}}(\mathbf{y}) = \quad (124)$$

$$\sum_{i=1}^N \ln \left[ \frac{P_{D,i} + (1 - P_{D,i}) \exp\left(-\frac{4}{\sigma^2} \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \hat{\mathbf{P}}\mathbf{h}_{i,j})\right)}{P_{F,i} + (1 - P_{F,i}) \exp\left(-\frac{4}{\sigma^2} \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \hat{\mathbf{P}}\mathbf{h}_{i,j})\right)} \right],$$

where above equation is obtained using the PDFs  $p(\mathbf{y}_i | \hat{\mathbf{H}}_i | \mathbf{U}_i = \mathbf{P}) \sim \sum_{j=1}^{N_f} \mathcal{CN}(\mathbf{P}\hat{\mathbf{h}}_{i,j}, \sigma^2 \mathbf{I}_L)$ ,  $p(\mathbf{y}_i | \hat{\mathbf{H}}_i | \mathbf{U}_i = -\mathbf{P}) \sim \sum_{j=1}^{N_f} \mathcal{CN}(-\mathbf{P}\hat{\mathbf{h}}_{i,j}, \sigma^2 \mathbf{I}_L)$  in (39) considering antipodal signaling, i.e., transmitted matrix  $\mathbf{U}_i = \mathbf{P}$  or  $\mathbf{U}_i = -\mathbf{P}$  for the local decisions corresponding to the presence or absence of

the primary signal respectively. The test statistic above can be further approximated at low SNR as,

$$T_{\text{UA}}(\mathbf{y}) = \sum_{i=1}^N \left[ a_i \sum_{j=1}^{N_f} \Re(\mathbf{y}_{i,j}^H \hat{\mathbf{P}}\mathbf{h}_{i,j}) \right]. \quad (125)$$

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