RIDESHARING: Fair Cost Distribution

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by

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under the guidance of

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CERTIFICATE OF APPROVAL

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This is to certify that the thesis titled **RIDESHARING: Fair Cost Distribution**, submitted by **SHASHANK SETHI**, to the Indian Institute of Technolog, Mandi, is a record of bonafide work under my (our) supervision and is worthy of consideration for the award of the degree of **B. Tech** of the Institute.

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Shashank Sethi

Abstract

Ridesharing refers to the sharing of vehicles by different passengers. It reduces vehicle trips, traffic congestion, and automobile emission. This makes ridesharing highly desirable in terms of economic and environmental sustainability. Our focus is on real-time ridesharing, in which passengers are matched using a mobile or web application to participate in ridesharing. More specifically, how the cost should be distributed among them, when they do so.

The notion of Sequential Individual Rationality (SIR) is used to characterize the routes for 'fair' cost distribution. However, this work is not focused on the routing aspect of ridesharing, but on how the cost should be distributed among the passengers participating in ridesharing.

The first case comprises using an existing cost sharing framework that uses SIR for single drop-off scenario. This framework is extended for single-pickup, multiple drop-off scenario. The effectiveness of the framework in these scenarios is evident by simulating multiple instances of ridesharing on a grid-like structure that comprises the passenger space.

Finally, the framework is extended for multiple-pickup, multiple-dropoff scenario. And the effectiveness is shown, not by means of mathematical proofs, but by using multiple instances of simulations.

There might still be a lot of scope of refinement in the proposed scheme, but that is beyond the scope of this work.

Contents

Abstract **ii**

Abbreviations **v**

List of Symbols **vi**

List of Figures **1**

1 Introduction **2**

1.1 Objective 3

2 Background and Related Work **4**

2.1 Model for Cost Sharing in Ridesharing 4

2.1.0.1 Cost Functions 4

2.1.0.2 Cost Sharing Scheme 5

2.1.0.3 Disutility and Individual Rationality 5

2.1.0.4 Sequential Individual Rationality 6

3 Solution Methodology **7**

3.1 Multiple-Pickup Single-Drop (MP-SD) scenario 7

3.1.1 The distance and cost functions 7

3.1.2 Cost Sharing Scheme : MP-SD 8

3.2 Single-Pickup Multiple-Drop (SP-MD) scenario 9

3.2.1 The distance and cost functions 9

3.2.2 Cost Sharing Scheme : SP-MD 9

3.3 Multiple-Pickup Multiple-Drop (MP-MD) scenario 10
3.3.1 Sub-paths: MP-SD and SP-MD 10
3.3.2 The disutility of passengers 11
3.3.3 Cost Sharing Scheme : MP-MD 11

4 Experimental Studies and Results 13
4.1 Matching Algorithm 14
4.2 Experimental Findings 15
4.2.1 Multiple-Pickup Single-Drop scenario (MP-SD) 15
4.2.1.1 Cost Scheme Comparison: MP-SD 17
4.2.2 Single-Pickup Multiple-Drop scenario (SP-MD) 19
4.2.2.1 Cost Scheme Comparison: SP-MD 19
4.2.3 Multiple-Pickup Multiple-Drop scenario (MP-MD) 20
4.2.3.1 Cost Scheme Comparison: MP-MD 20

5 Conclusion and Future Work 24
5.1 Conclusion 24
5.2 Future Work 24

References 25

Abbreviations

IR	- Individual Rationality
SIR	- Sequential Individual Rationality
MP-SD	- Multiple-Pickup Single-Drop
SP-MD	- Single-Pickup Multiple-Drop
MP-MD	- Multiple-Pickup Multiple-Drop

Symbols

OC	- Inconvenience Cost
IC	- Inconvenience Cost
DU	- Disutility
f	- Cost share

List of Figures

4.1 SIR feasible region for first passenger 15

4.2 SIR feasible region for second passenger 16

4.3 SIR feasible region for third passenger 16

4.4 SIR feasible region for fourth passenger 17

4.5 Normalized cost shares for cost sharing scheme with and without internal transfers for MP-SD scenario 18

4.6 Frequency of No. of SIR feasible paths obtained for MP-SD for 100 instances 18

4.7 Normalized cost shares for cost sharing scheme with and without internal transfers for SP-MD scenario (for routes with 4 passengers) 19

4.8 Frequency of No. of SIR feasible paths obtained for SP-MD for 100 instances 20

4.9 Normalized cost shares for cost sharing scheme with and without internal transfers for MP-MD scenario ((for routes with 3 passengers) 21

4.10 Normalized cost shares for cost sharing scheme with and without internal transfers for MP-MD scenario (for routes with 4 passengers) 22

4.11 Frequency of No. of SIR feasible paths obtained for MP-MD for 200 instances 22

Chapter 1

Introduction

Commercial ridesharing has been widely adopted in most nations. Despite the inconvenience one faces due to ridesharing, it is still a preferable mode of transport as it greatly reduces the cost of travel. In fact, the major motivation for cab companies to provide ridesharing services can be attributed to the fact that ridesharing leads to cost saving for the service consumers.

Even though the benefits of ridesharing are good for the society, many users are reluctant about it, as there can be some concerns about safety, privacy and most importantly the delays due to detours that cause inconvenience to the ridesharing passengers.

Presently, in India, ridesharing services are being provided by OLA and UBER in the name of OLAShare and UberPool, respectively. Ridesharing in OLA is a prepaid service, which means that the length and duration of the detours that a passenger undergoes, are not directly taken into consideration while computing the cost of such a ride.

Hence, there is a lot of scope of improvement in the way cost is distributed among passengers participating in ridesharing.

1.1 Objective

The aim of this work is to come up with an efficient cost sharing scheme for real-time ridesharing. And to check its effectiveness using appropriate trial runs or simulations.

The current cost sharing scheme does not directly take into consideration, the inconvenience caused to passengers due to pickups and dropoffs of other passengers. A traditional scheme would simply split the cost among the passengers traveling a particular segment in ridesharing. However, we can incorporate the inconveniences by requiring that passengers compensate each other in accordance with the inconvenience that they cause or is caused to them. Using the concept of 'disutility' and SIR, we can ensure that passengers are better off after any subsequent passenger joins. As we will see, this is the defining criteria for SIR.

Chapter 2

Background and Related Work

The notion of SIR has been introduced in the work of Gopalakrishnan, R. - *The Costs and Benefits of Ridesharing: Sequential Individual Rationality and Sequential Fairness* [1]

2.1 Model for Cost Sharing in Ridesharing

This model for cost sharing explicitly takes into account the inconvenience costs experienced by passengers due to detours they endure as a result of other passengers being picked up and dropped off.

Let $\mathcal{N}$ ={1, 2, ..., n} denote the set of passengers. For each passenger $i \in \mathcal{N}$, let S_i and D_i denote their pickup and dropoff points, which can be assumed to belong to an underlying metric space.

2.1.0.1 Cost Functions

We define the following cost functions:

(1) Operational Cost: The operational cost of a ride involving a set of passengers $S \subseteq \mathcal{N}$, otherwise known as the meter fare, is defined as

$$OC(S\,; r_S) = \alpha_{op} d(S\,; rS) \tag{2.1}$$

where $d(S\,; r_S)$ the total distance traveled along route r_S.

(2) Inconvenience Costs: In a ride involving a set of passengers S N, for each $i \in \mathcal{S}$, the inconvenience cost incurred due to other passengers is defined as

$$\mathcal{IC}_i(S\,; r_S) = \alpha_i(d_i(S\,; r_S) - d_i(i; r_{\{i\}})) \tag{2.2}$$

where $d_i(S\,; r_S)$ is the distance traveled along route r_S from S_i to D_i.

Here, $\alpha_{op} > 0$ is the price (in commercial ridesharing) per unit distance, and, for each passenger $i \in \mathcal{S}$, $\alpha_i \geq 0$ is the inconvenience cost of i per unit distance.

2.1.0.2 Cost Sharing Scheme

A cost sharing scheme f is a function that specifies, for any subset $S \subseteq \mathcal{N}$, how $\mathcal{OC}(S\,; r_S)$ is distributed among the passengers in S. That is, $f(i; S\,; r_S)$ denotes the portion of $\mathcal{OC}(S\,; r_S)$ allocated to passenger $i \in \mathcal{S}$. We set $f(i; S\,; r_S) = 0$ whenever $i \notin S$.

A cost sharing scheme f is budget-balanced if it exactly recovers the operational cost of a ride, that is,

$$\sum_{i \in \mathcal{S}} f(i; S\,; r_S) = \mathcal{OC}(S\,; r_S) \qquad \forall \quad S \subseteq \mathcal{N}. \tag{2.3}$$

2.1.0.3 Disutility and Individual Rationality

When a subset of passengers $S \subseteq \mathcal{N}$ share a ride, the disutility to a passenger $i \in \mathcal{S}$ is defined as the sum of their monetary payment for the ride and their inconvenience cost due to any detours, that is,

$$\mathcal{DU}_i(S\,; r_S) = f(i; S\,; r_S) + \mathcal{IC}_i(S\,; r_S) \tag{2.4}$$

A cost sharing scheme f is individually rational (IR) on route $r_{\mathcal{N}}$ if

$$\mathcal{DU}_i(N; r_N) \leq \mathcal{DU}_i(\{i\}; r_{\{i\}}) \qquad \forall \quad i \in \mathcal{N}. \tag{2.5}$$

A route $r_{\mathcal{N}}$ is IR-feasible if there exists a budget-balanced cost sharing scheme f that is

IR on $r_{\mathcal{N}}$.

2.1.0.4 Sequential Individual Rationality

Individual rationality (IR) requires that passengers are better off ridesharing than not, it takes into account their disutilities at the end of their ride. That is, the disutility of each passenger should decrease when the whole ride ends. In that sense, it is a static property. However, the passengers face inconvenience during the ride, when the vehicle undertakes detours to pick up and drop off other passengers. So, a stronger property that requires that IR hold at every stage of the ride, that is, every time a new passenger is picked up. This property has been called sequential IR (SIR).

Let $i \in \mathcal{N}$ be the i-th passenger to be picked up, in the route $r_{\mathcal{N}}$. Let $\mathcal{T} = \{t_1, t_2, ..., t_n\}$ denote the set of pickup times, that is, t_i denotes the time at which i-th passenger is picked up according to route $r_{\mathcal{N}}$. Let S(i) = {1,2,...,i} denote the set of passengers that have been picked up upto time t_i. Route $r_{\mathcal{N}}\ t$ denotes the route identical to $r_{\mathcal{N}}$ up to time t, but thereafter does not pick up any more passengers, and proceeding only to drop the remaining passengers.

The disutility of a passenger $i \in \mathcal{N}$ at time $t_j \in \mathcal{T}$ is defined as

$$\mathcal{DU}_i(t_j) = \begin{cases} \mathcal{DU}_i(\{S(j); r_{\mathcal{N}}(t_j)\} & \forall \quad i \in S(j). \\ \mathcal{DU}_i(\{i\}; r_{\{i\}}) & otherwise \end{cases} \tag{2.6}$$

A cost sharing scheme f is sequentially individually rational (SIR) on route $r_{\mathcal{N}}$ if

$$\mathcal{DU}_i(t_j) \leq \mathcal{DU}_i(t_{j-1}) \qquad \forall \quad 2 \leq j \leq n \quad \forall \quad i \in \mathcal{N}. \tag{2.7}$$

A route $r_{\mathcal{N}}$ is SIR-feasible if there exists a budget-balanced cost sharing scheme f that is SIR on $r_{\mathcal{N}}$.

[1].

Chapter 3

Solution Methodology

We extend the cost sharing framework to incorporate SIR in more practical scenarios of ridesharing - Single-Pickup Multiple-Drop, SP-MD (the airport pickup scenario), and Multiple-Pickup Multiple-Drop, MP-MD (the typical rideshare scenario). Also, we test effectiveness of a new cost sharing scheme that takes into account the inconvenience that the passengers face due to the detours they face as a result of pickups and drop-offs of other passengers, for each of these scenarios.

3.1 Multiple-Pickup Single-Drop (MP-SD) scenario

The distance and cost functions mentioned in previous chapter can be described for MP-MD scenario as follows:

3.1.1 The distance and cost functions

$S(j) = \{1,2,...,j\}$ denotes the set of passengers that are picked up till time t_j. The distance functions would become

$$d(S(j)) = \sum_{k=1}^{j-1} S_k S_{k+1} + S_j D, \quad 1 \leq j \leq n. \tag{3.1}$$

$$d_i(S(j)) = \sum_{k=i}^{j-1} S_k S_{k+1} + S_j D, \quad 1 \le i \le j \le n. \tag{3.2}$$

The cost functions become

$$\mathcal{OC}(S; r_S) = \alpha_{op} d(S(j)) = \alpha_{op}\Big(\sum_{k=1}^{j-1} S_k S_{k+1} + S_j D\Big), \quad 1 \le j \le n. \tag{3.3}$$

$$\mathcal{IC}_i(S(j)) = \alpha_i(d_i(S(j)) - d_i(i)) = \alpha_i\Big(\sum_{k=i}^{j-1} S_k S_{k+1} + S_j D - S_i D\Big), \quad 1 \le i \le j \le n. \tag{3.4}$$

The disutilities are given by

$$\mathcal{DU}_i(S(j)) = f(i, S(j)) + \alpha_i\Big(\sum_{k=i}^{j-1} S_k S_{k+1} + S_j D - S_i D\Big), \quad 1 \le i \le j \le n. \tag{3.5}$$

3.1.2 Cost Sharing Scheme : MP-SD

A fair cost sharing scheme needs the consider the inconvenience that passengers face due to rideshare, by facilitating internal transfer of cost between passengers based on the degree of inconvenience that is caused to the passenger, and by the passenger.

1. Distributing the operational cost of each segment, among the ridesharing passengers in that segment, equally.

$$f_1(i, S(j)) = \alpha_{op}\Big(\sum_{k=i+1}^{j} \frac{S_{k-1}S_k}{k-1} + \frac{S_j D}{j}\Big), \quad 1 \le i \le j \le n. \tag{3.6}$$

2. A passenger i, compensating all $'i-1'$ passengers due to the detour it takes.

$$f_2(i, S(j)) = \alpha_i(i-1)(S_{i-1}S_i + S_i D - S_{i-1}D), \quad 1 \le i \le j \le n. \tag{3.7}$$

3. The compensation that the i-th passenger receives due to the inconvenience caused to it.

$$f_3(i, S(j)) = \sum_{k=i+1}^{j} \alpha_k(S_{k-1}S_k + S_k D - S_{k-1}D), \quad 1 \le i \le j \le n. \tag{3.8}$$

A cost sharing scheme that takes into account the above components:

$$f(i, S(j)) = f_1(i, S(j)) + f_2(i, S(j)) - f_3(i, S(j)), \quad 1 \leq i \leq j \leq n. \tag{3.9}$$

3.2 Single-Pickup Multiple-Drop (SP-MD) scenario

The distance and cost functions for the MP-MD scenario can be summarized as follows:

3.2.1 The distance and cost functions

$D(j)$ = {1,2,...,j} denotes the set of passengers that are to be picked up from a single pickup point denoted by {S}. Let $\{D_1, D_2, ..., D_j\}$ denotes the set of drop locations for each of the picked up passenger. The distance functions become:

$$d(D(j)) = SD_1 + \sum_{k=1}^{j-1} D_k D_{k+1}, \quad 1 \leq j \leq n. \tag{3.10}$$

$$d_i(D(j)) = SD_1 + \sum_{k=1}^{i-1} D_k D_{k+1}, \quad 1 \leq i \leq j \leq n. \tag{3.11}$$

The cost functions become

$$\mathcal{OC}(D; r_D) = \alpha_{op} d(D(j)) = \alpha_{op}\Big(SD_1 + \sum_{k=1}^{i-1} D_k D_{k+1}\Big), \quad 1 \leq j \leq n. \tag{3.12}$$

$$\mathcal{IC}_i(D(j)) = \alpha_i(d_i(D(j)) - d_i(i)) = \alpha_i\Big(SD_1 + \sum_{k=1}^{i-1} D_k D_{k+1} - SD_i\Big), \quad 1 \leq i \leq j \leq n. \tag{3.13}$$

The disutilities are given by

$$\mathcal{DU}_i(D(j)) = f(i, D(j)) + \alpha_i\Big(SD_1 + \sum_{k=1}^{i-1} D_k D_{k+1} - SD_i\Big), \quad 1 \leq i \leq j \leq n. \tag{3.14}$$

3.2.2 Cost Sharing Scheme : SP-MD

A fair cost sharing scheme similar to the scheme for MP-SD is described as follows:

1. Distributing the operational cost of each segment, among the ridesharing passengers in that segment, equally.

$$f_1(i, D(j)) = \alpha_{op}\Big(\frac{SD_1}{j} + \sum_{k=1}^{i-1} \frac{D_k D_{k+1}}{j-k}\Big), \quad 1 \le i \le j \le n. \tag{3.15}$$

2. A passenger i, compensating all $'j - i'$ passengers due to the detour taken for it takes.

$$f_2(i, D(j)) = \alpha_i(j-i)(SD_i + D_i D_{i+1} - SD_{i+1}), \quad 1 \le i \le j \le n. \tag{3.16}$$

3. The compensation that the i-th passenger receives due to the inconvenience caused to it.

$$f_3(i, D(j)) = \sum_{k=1}^{i-1} \alpha_k(SD_k + D_k D_{k+1} - SD_{k+1}), \quad 1 \le i \le j \le n. \tag{3.17}$$

A cost sharing scheme for SP-MD scenario becomes:

$$f(i, D(j)) = f_1(i, D(j)) + f_2(i, D(j)) - f_3(i, D(j)), \quad 1 \le i \le j \le n. \tag{3.18}$$

3.3 Multiple-Pickup Multiple-Drop (MP-MD) scenario

We handle the problem of MP-MD by breaking it into two relatively simpler sub-problems: MP-SD and SP-MD. As a result, we can analyse the SIR feasibilty of these two sub-paths, the framework for which has been described in this previous sections. The motivation for such an approach lies in the fact that the inconvenience caused by pickups and drops of passenger need to be handled separately.

3.3.1 Sub-paths: MP-SD and SP-MD

The MP-MD scenario consists of sets of pickups and drops: $\{S_1, S_2, ..., S_j\}$ and $\{D_1, D_2, ..., D_j\}$.

1. **MP-SD subpath:** This subpath will include the pickups of all the passengers, with their common destination being the first drop that will happen. This drop need not be the drop of first passenger, and we represent it as $\{D_x^1\}$, $x \in \{1, 2, ..., j\}$

where x denotes the passenger whose drop it is, and the superscript denotes the order of the drop.

Hence we check the SIR feasibility of MP-SD subpath represented by $\{S_1, S_2, ..., S_j\}$ and $\{D_x^1\}$. And compute the cost shares for each passenger, in this subpath.

2. **SP-MD subpath:** This subpath will include the drop-offs of all the passengers, with their common pickup point being the last pickup that will happen. This subpath comprises of pickup $\{S_j\}$, and drop locations $\{D_1, D_2, ..., D_j\}$.

   Next, we check the SIR feasibility of SP-MD subpath represented by $\{S_j\}$ and $\{D_{x_1}^1, D_{x_2}^2, ..., D_{x_j}^j\}$. And compute the cost shares for each passenger in this subpath, omitting the cost share computation for first segment $S_j D_{x_1}^1$, as it has already been accounted for in previous subpath.

3.3.2 The disutility of passengers

In case of MP-MD, the disutility of a passenger at any stage t_j of rideshare can be given by the sum of disutilities of the passenger in the subpaths MP-SD and SP-MD, at that stage

$$\mathcal{DU}_i(t_j) = \mathcal{DU}_i(S(j); r_S(t_j)) + \mathcal{DU}_i(D(j); r_D(t_j)) \quad i \in S(j) \tag{3.19}$$

SIR requires that the disutility of each passenger should decrease at every stage of ridesharing, i.e. whenever a new passenger joins. This means that the disutilities of the passengers decrease in the two subpaths upon addition of a passenger in ridesharing. It follows that their sum would also decrease at every stage. Hence, this choice of representing disutilities in this manner is consistent with the necessary condition for SIR feasibilty.

3.3.3 Cost Sharing Scheme : MP-MD

The cost shares of the passengers can be obtained by adding their shares in the two subpaths, and handling the operational cost of the segment which is common to both subpaths, only once.

1. Distributing the operational cost of each segment, among the ridesharing passengers in that segment, equally.

$$f_1(i, t_j) = f_1(i, S(j)) + f_1(i, D(j)) - \alpha_{op}\left(\frac{S_j D^1_{x_1}}{j}\right), \quad 1 \leq i \leq j \leq n. \tag{3.20}$$

2. A passenger i, compensating all $'j - i'$ passengers due to the detour taken for it takes.

$$f_2(i, t_j) = f_2(i, S(j)) + f_2(i, D(j)), \quad 1 \leq i \leq j \leq n. \tag{3.21}$$

3. The compensation that the i-th passenger receives due to the inconvenience caused to it.

$$f_3(i, t_j) = f_3(i, S(j)) + f_3(i, D(j)), \quad 1 \leq i \leq j \leq n. \tag{3.22}$$

Overall cost shares for $i^{(}th)$ passenger at the stage in which j passengers participate in ridesharing:

$$f(i, t_j) = f_1(i, t_j) + f_2(i, t_j) - f_3(i, t_j), \quad 1 \leq i \leq j \leq n. \tag{3.23}$$

Chapter 4

Experimental Studies and Results

We present the results of simulations run to check the effectiveness of cost scheme in each of the aforementioned scenarios - MP-SD, SP-MD, and MP-MD, respectively. By simulating multiple instances of ridesharing on a grid-like structure that comprises the passenger space, we could extract insights from the data generated. This would reflect how the proposed so-called fair cost sharing scheme differs from the traditional way of splitting costs.

An arbitrary routing algorithm was used to match passengers from a passenger space of 500 randomly generated passengers. The resulting SIR feasible routes comprise one instance of such simulation. The plots used to show our findings show the normalized cost shares at the final stage of rideshare of first, second, third and fourth passengers, averaged over data collected from 1000 instances of simulations in each scenario.

4.1 Matching Algorithm

The choice of routing algorithm is irrelevant, as we are not concerned with the routing aspect of ridesharing. Any routing algorithm can accomodate the cost sharing framework which we have built upon in this work. However, a typical routing algorithm would match the passengers for ridesharing, plan a route, and also try to maximize the operator's revenue. The flow of the matching algorithm that we used in simulations is summarized below:

1. start from every passenger as a source passenger.

2. try all passengers as potential next passenger. Selecting the one which gives SIR feasible path till that point. In case there is more that one possible passenger which satisfies this criteria, choose the one that causes minimum inconvenience to the previous passenger, that is minimizing the Inconvenience Cost of previous passenger.

3. repeat step (2) until 4 passengers have been picked up, or no passenger satisfies the constraints for a potential next passenger.

4. The next step is to sort the SIR feasible routes, such that routes with maximum number of passengers are desired so these will be chosen first. To compare routes that have same number of passengers, choose the one with higher operational cost, to maximize the service providers revenue.

5. Remove the passengers of the most optimal route from the passenger space, and start from step (1)

4.2 Experimental Findings

4.2.1 Multiple-Pickup Single-Drop scenario (MP-SD)

For single drop-off scenario, we can visualize the shape the SIR feasible region - the region from which a passenger can be chosen so as to get a path which would satisfy the SIR criterion.

Considering the following SIR feasible route, we obtain the SIR feasible regions for each passenger.

Route: $(62, 10) \rightarrow (49, 11) \rightarrow (33, 10) \rightarrow (29, 10) \rightarrow (9, 12)$

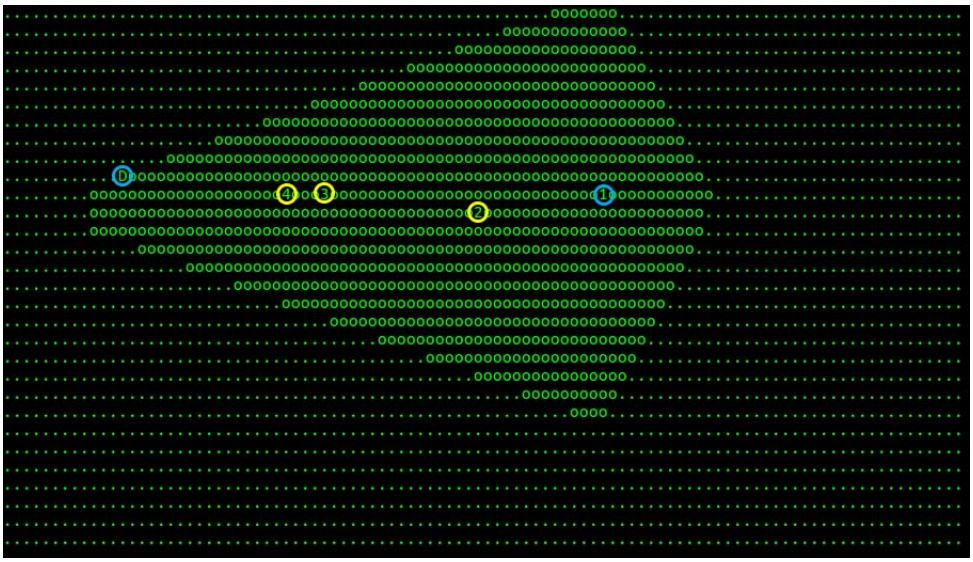

Fig. 4.1: SIR feasible region for first passenger

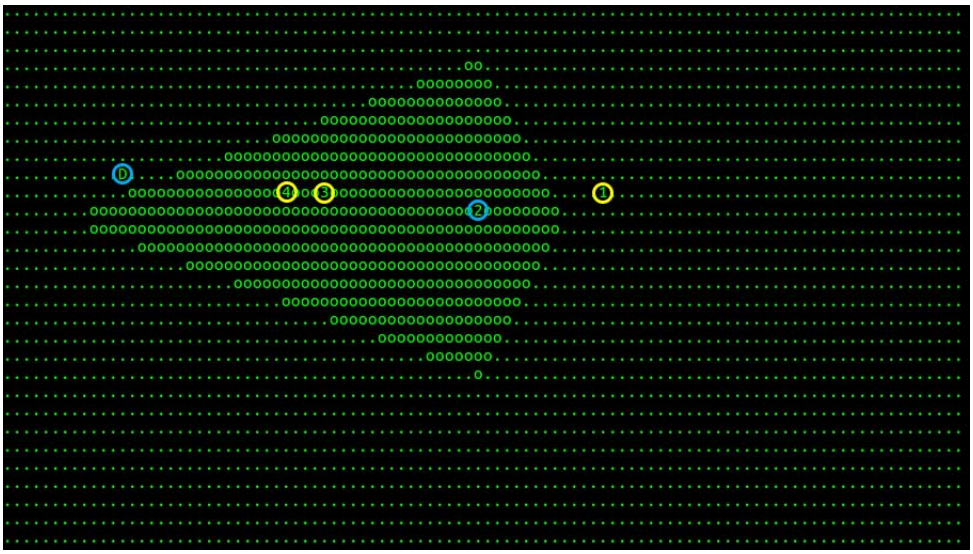

Fig. 4.2: SIR feasible region for second passenger

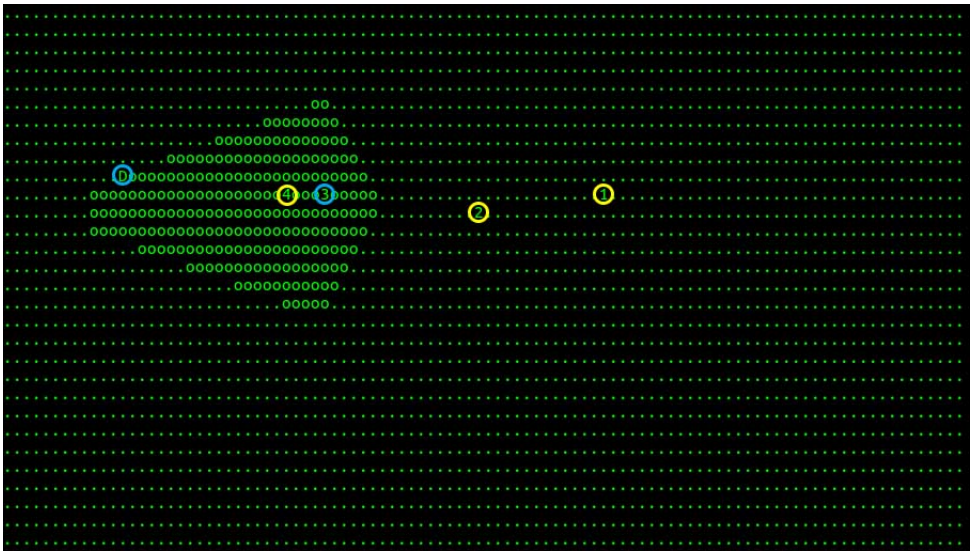

Fig. 4.3: SIR feasible region for third passenger

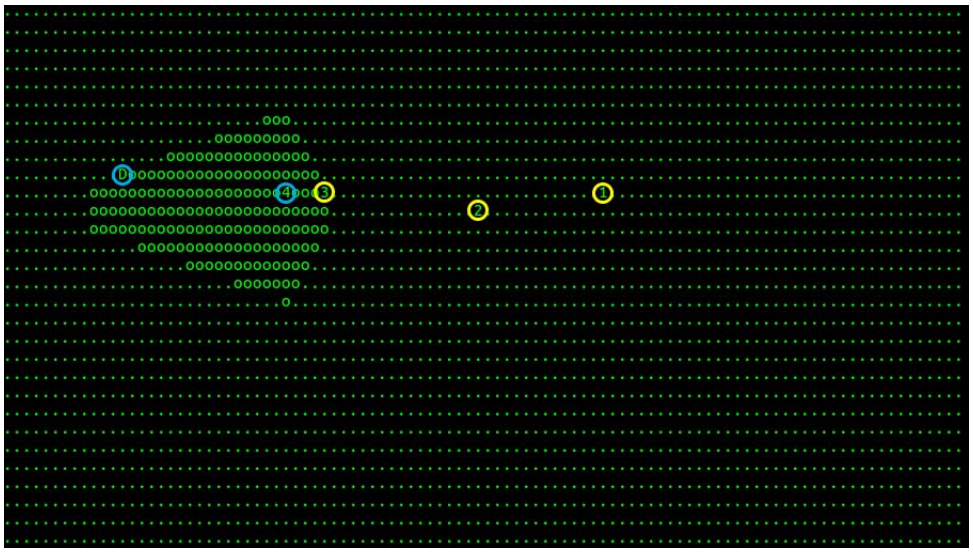

Fig. 4.4: SIR feasible region for fourth passenger

4.2.1.1 Cost Scheme Comparison: MP-SD

Our cost scheme incorporates internal transfers between passengers, but in case of MP-SD only the initial passengers face inconvenience due to subsequent detours and hence they are compensated for. So, naturally, the initial passengers will be better off with the internal transfers, and passengers who join later will not be. This result is captured in the plot that follows.

Recall from eq (3.9) and (3.6) that $f(i, S(j))$ represents the cost share when internal transfer of compensations is taken into consideration, and $f_1(i, S(j))$ represents the cost shares without internal transfers. These are the two schemes that we are comparing. For both the new and the traditional scheme, the costs shares for each passenger are normalized using the operational cost of the respective passenger that he would have paid if he did not participate in ridesharing, and travelled solo. The averages of the normalized cost shares are depicted in the plot.

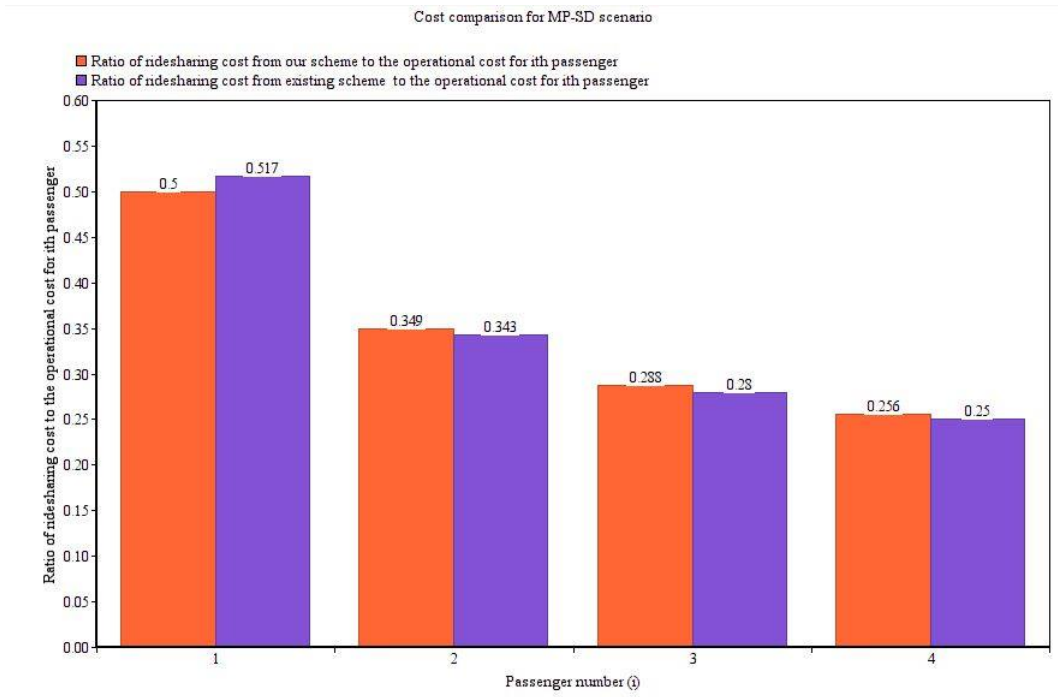

Fig. 4.5: Normalized cost shares for cost sharing scheme with and without internal transfers for MP-SD scenario

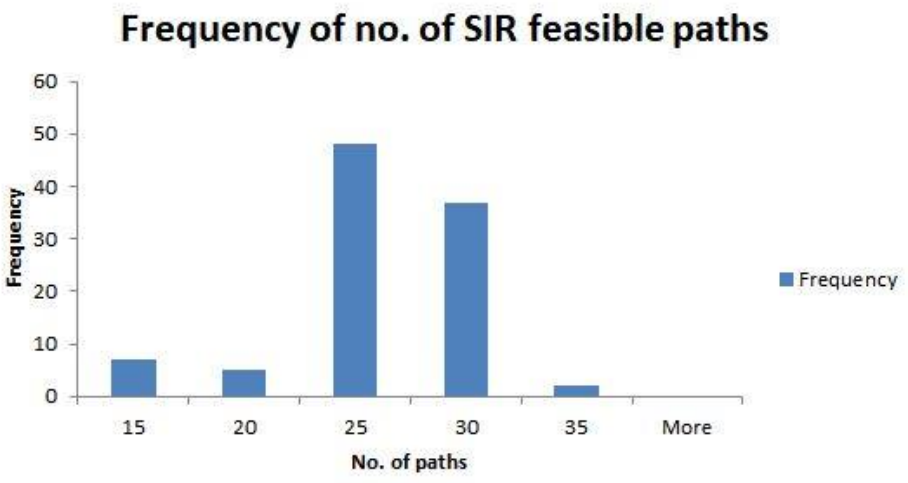

Fig. 4.6: Frequency of No. of SIR feasible paths obtained for MP-SD for 100 instances

The above histogram shows that for 100 instances, most of the times 25 SIR feasible paths were obtained. So 1000 instances imply that at most 25,000 paths were considered while averaging the normalized cost shares.

4.2.2 Single-Pickup Multiple-Drop scenario (SP-MD)

4.2.2.1 Cost Scheme Comparison: SP-MD

In case of SP-MD only the passengers who are dropped later, face inconvenience due to detours taken because of initial dropoffs. Hence the later passengers are compensated for, and those passengers will be better off with the internal transfers, and passengers who are dropped initially will not be. This result is captured in the plot that follows.

Recall from eq (3.18) and (3.15) that $f(i, D(j))$ represents the cost share when internal transfer of compensations is taken into consideration, and $f_1(i, D(j))$ represents the cost shares without internal transfers. These are the two schemes that we are comparing. For both the new and the traditional scheme, the costs shares for each passenger are normalized using the operational cost of the respective passenger that he would have paid if he did not participate in ridesharing, and travelled solo. The averages of the normalized cost shares are depicted in the plot.

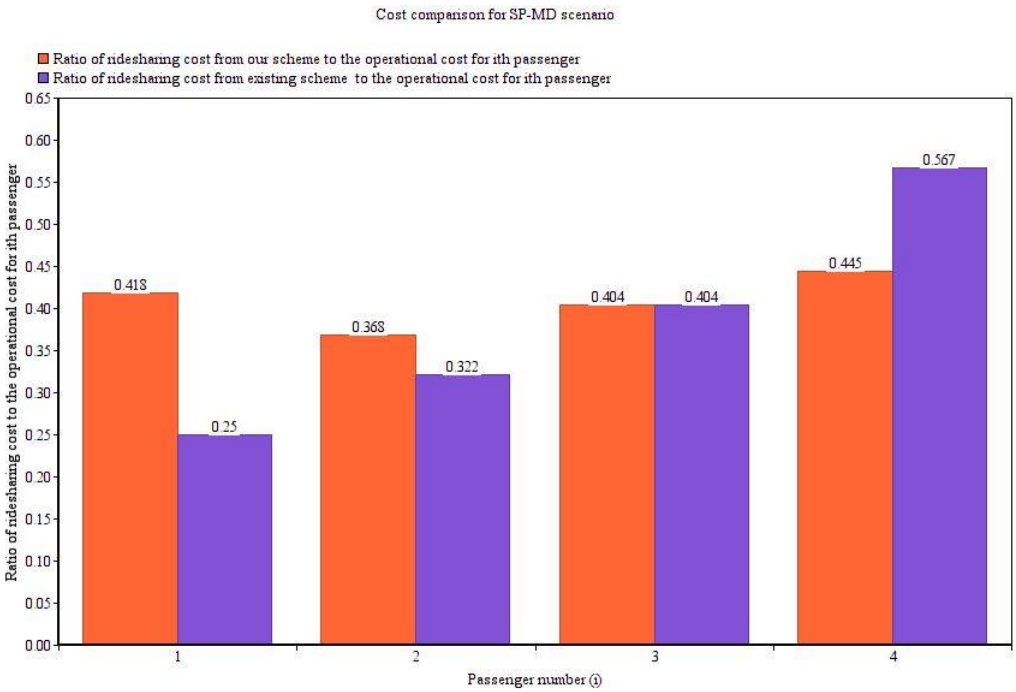

Fig. 4.7: Normalized cost shares for cost sharing scheme with and without internal transfers for SP-MD scenario (for routes with 4 passengers)

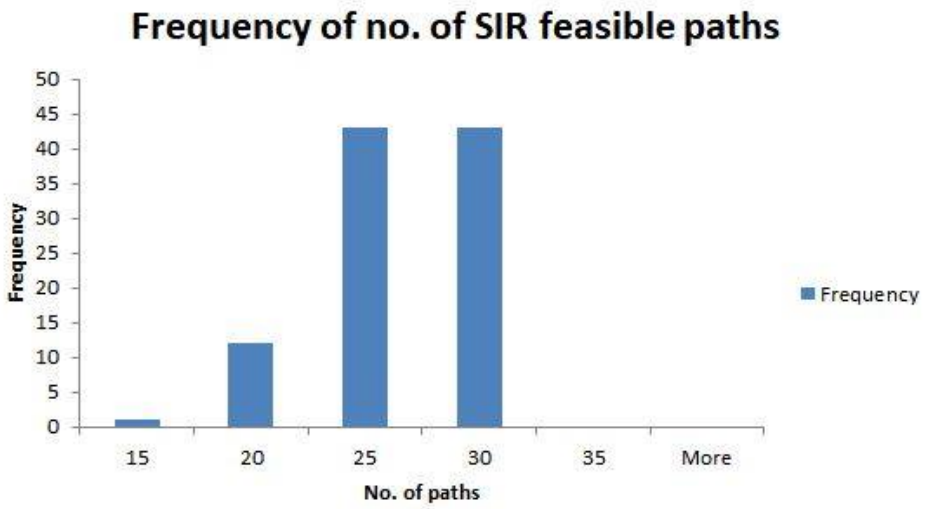

Fig. 4.8: Frequency of No. of SIR feasible paths obtained for SP-MD for 100 instances

The above histogram shows that for 100 instances, most of the times nearly 27 SIR feasible paths were obtained. So 1000 instances imply that at most 27,000 paths were considered while averaging the normalized cost shares.

4.2.3 Multiple-Pickup Multiple-Drop scenario (MP-MD)

4.2.3.1 Cost Scheme Comparison: MP-MD

In case of MP-MD, initial passengers face incovenience due to subsequent pick-ups and cause inconvenience due to their dropoffs. So the pattern in which some passengers are favored by the scheme should not be observed here. And the scheme would, in fact, quantify the inconveniences and reflect it through the internal transfers. Hence the results should be different from the traditional scheme which does not consider the inconveniences, but which passengers pay more, and which one's pay less would purely be determined by the situation at hand. And that is the main idea behind having a fair scheme.

Recall from eq (3.23) and (3.20) that $f(i)$ represents the cost share when internal transfer of compensations is taken into consideration, and $f_1(i)$ represents the cost shares without internal transfers. These are the two schemes that we are comparing. For both the new and the traditional scheme, the costs shares for each passenger are normalized using the opera-

tional cost of the respective passenger that (s)he would have paid if (s)he did not participate in ridesharing, and travelled solo. The averages of the normalized cost shares are depicted in the plot.

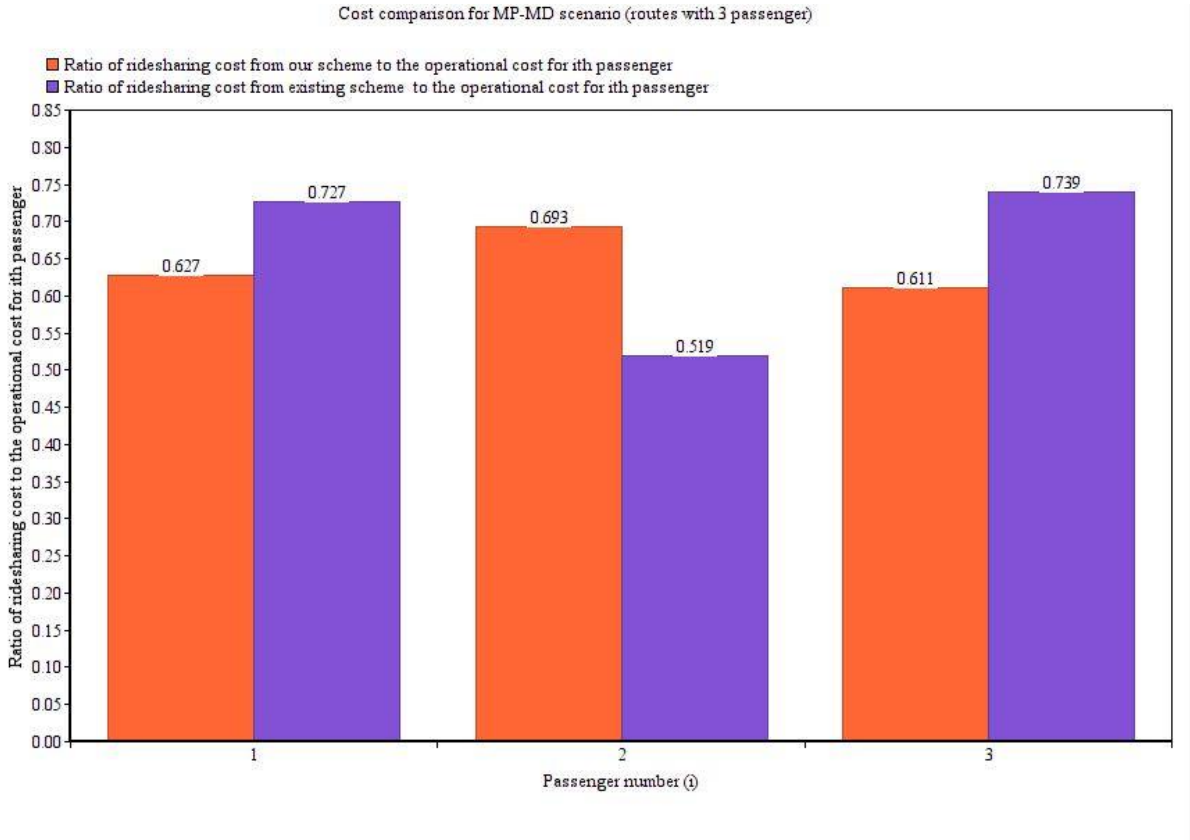

Fig. 4.9: Normalized cost shares for cost sharing scheme with and without internal transfers for MP-MD scenario ((for routes with 3 passengers)

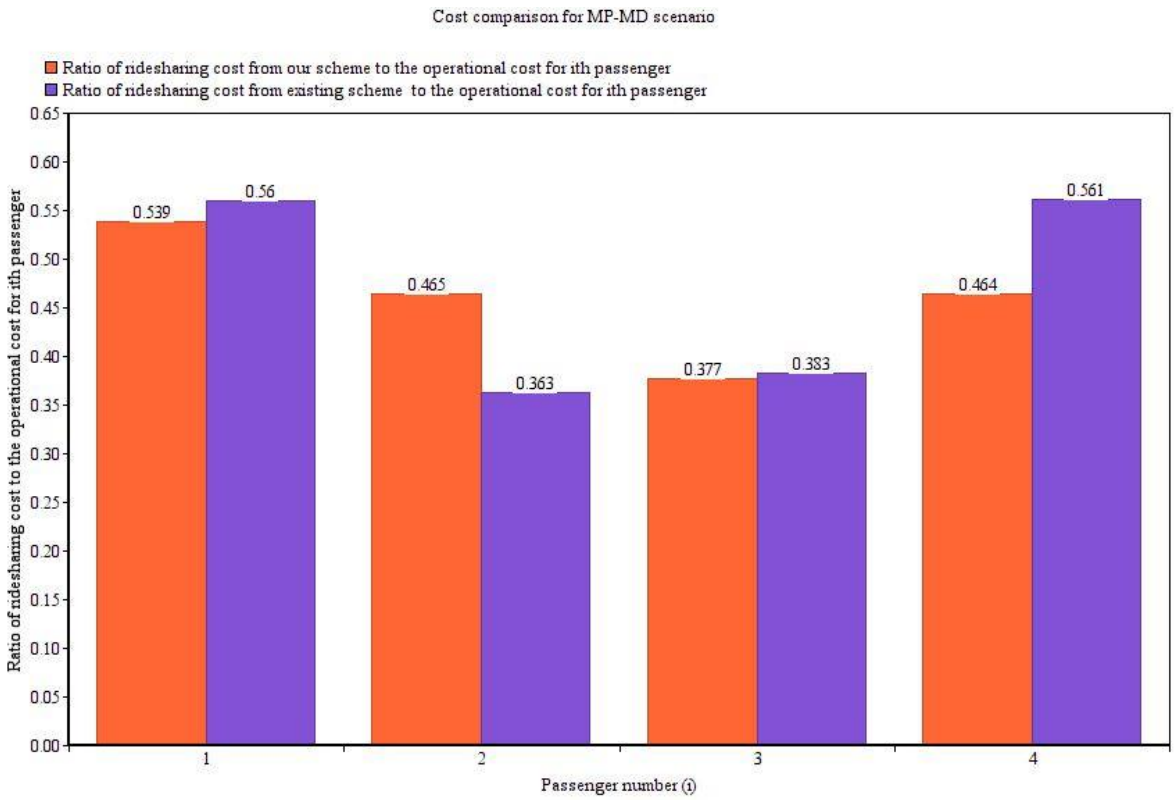

Fig. 4.10: Normalized cost shares for cost sharing scheme with and without internal transfers for MP-MD scenario (for routes with 4 passengers)

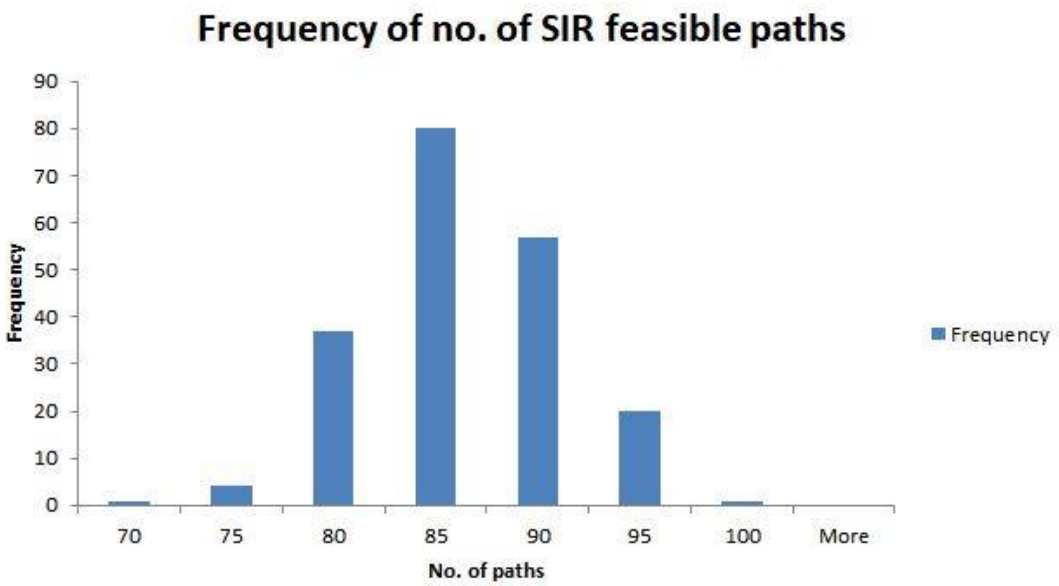

Fig. 4.11: Frequency of No. of SIR feasible paths obtained for MP-MD for 200 instances

The above histogram shows that for 200 instances, most of the times around 85 SIR feasible paths were obtained. So 1000 instances imply that at most 85,000 paths were considered while averaging the normalized cost shares.

Chapter 5

Conclusion and Future Work

5.1 Conclusion

From the findings in different aforementioned scenarios, it is evident that, with the new scheme the passengers might pay a little less, or more compared to the scheme that does not consider the inconvenience of passengers. However, it is a fair scheme as it quantifies what passengers should actually pay, based on the degree of inconvenience that they cause, or face.

5.2 Future Work

The share scheme can be worked upon further to refine the transfers that take between passengers due to inconvenience caused, or faced by the passengers. This can be done by modifying the choice of coefficients of the cost functions, and the cost share scheme. This would result in cost scheme being further optimized, and several factors that come into play in practical scenarios can be accounted for. The framework is open to extension, refinement, etc.

References

[1] R. Gopalkrishnan, "The costs and benefits of ridesharing: Sequential individual rationality and sequential fairness," Jul. 2016.

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