SUMMER INTERNSHIP PROJECT REPORT

ON

OPINION DYNAMICS

AN INTRODUCTION AND REVISITING SOME CLASSICAL RESULTS

SUBMITTED BY

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ACKNOWLEDGEMENT

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CONTENTS

1.INTRODUCTION

Opinion dynamics is the science that relates to the studying of opinion and behavior. Opinions are the factors that modulate human behavior, thus playing an essential role in many global challenges that the world and the society has been facing. Some of these challenges are global financial crisis, global pandemics, urbanization and migration patterns apart from climate and environmental change patterns[1].

Owing to such crucial and vast utility, it has drawn attention from social scientists as well as from mathematicians, physicists, and computer scientists, in the growing interdisciplinary field of complex system science[1].

Opinion formation is a complex process affected by the interplay of different interacting agents holding on to their own opinion as well as adding a negative or positive component that results from the communication between peers[2].

Several models have been developed to assist the understanding of the mechanism of the behavior of the particles and opinion formation. The modeling schemes may range from simple binary models to multi dimensional approach models.

2.MODEL UNDER CONSIDERATION-DEFFUANT ET AL'S MODEL

Deffuant et al. considered a model whereby the agents choose to adjust their opinions towards convergence if their difference lies within a particular threshold.

We consider a population of *pop* agents with opinion *X*. The agents randomly meet and choose to share their opinion if and only if the difference between their respective opinions is less than a particular threshold value *Th*[2].

Mathematically,
If the difference $diff = | x - x'|$ ---------- (i)
and $diff < Th$ ---------------- (ii)
then,

$$x = x + \beta (x' - x) \text{ ------ (iii)} \quad \text{and} \quad x' = x + \beta (x - x') \text{ ------(iv)}$$

where, β is the convergence parameter lying between 0 and 0.5 during simulations.
The reason for having a threshold value is that the opinions get shared only if they are close enough else the agents do not bother to discuss. The reason for such differences might be lack of understanding, conflicts or social pressure. Although we may not have such openness to discussion yet we consider the Th value to be constant throughout a given population.

3. ALGORITHM OF MATLAB SIMULATION

The above equations were simulated on MATLAB by considering a population of *pop* random variables having a uniform distribution between 0 and 1.

Algorithm:

- Start
- Set the Threshhold Th , Beta β and Population pop
- Set the *instance* value as to how many instances need to be run
- Assign empty Evolution E matrix
- for i = 1 to *instance*
- R = vector having *pop* random variables.
- Choose unique pairs
    - Difference matrix = skew symmetric matrix having the difference of all the possible pairs.
    - Find pairs whose difference < Threshold Th
    - Find a unique random pair
- Update entries after opinion sharing according to equations (iii) and (iv).
- if 2-norm of histogram of previous row and current row <= convergence criteria , then current row is instant of convergence .
- Number of peaks = Find the number of non zero entries in the histogram of the last row of E matrix .
- Save E matrix
- Repeat until i = *instances*
- Load the saved E matrix to plot or for other analyses.

Refer to Appendix I for code.

4. OBSERVATIONS

The code corresponding to this model was run several times and results were plotted for a population of 1000. Considering all the β values for a particular threshold, i.e. 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, we end up with 120 readings for a single *Th* value.

The convergence time was found out by checking the 2-norm of the difference of histogram of the previous updated entries and the histogram of the current update with respect to convergence criteria chosen to be 0.0001. It is important to note here that the convergence time does not correspond to real time. Every row of the Evolution Matrix is considered as an individual time instant.

TABLE 1.1 Number of Peaks (*P*) Corresponding to Threshold value (*Th*).

Each Threshold value was run for six different values of B which are 0.5, 0.4, 0.3, 0.2, 0.1 and 0.05. Each case was run for 20 different instances. Hence we land with 120 observations for each Threshold value. The dashes denote that that many numbers of peaks were not obtained at that instance.

Th /P	1	2	3	4	5	6	7	8	9	10	11

0.05	-	-	-	-	-	-	2	22	56	27	5
0.1	-	-	2	56	61	1	-	-	-	-	-
0.2	-	115	5	-	-	-	-	-	-	-	-
0.3	120	-	-	-	-	-	-	-	-	-	-
0.4	120	-	-	-	-	-	-	-	-	-	-
0.5	120	-	-	-	-	-	-	-	-	-	-

TABLE 1.2 Summary of Convergence Time and Number of peaks

The convergence criterion is chosen to be the norm of difference of the histograms of the previous entry with the current entry.

Sl no	Th	β	Convergence Time			Number of Peaks		
			Average	Minimum	Maximum	Average	Minimum	Maximum
1	0.05	0.05	116.29	86	144	8.21	7	9
2	0.05	0.1	67.1	54	89	9	8	10
3	0.05	0.2	40.2	30	60	9.2	8	11
4	0.05	0.3	16.9	8	26	9.7	8	11
5	0.05	0.4	27.5	19	44	9.15	8	10
6	0.05	0.5	26.3	19	35	9.1	7	10
7	0.1	0.05	140.5	94	231	4.25	3	5
8	0.1	0.1	70.15	53	95	4.75	4	5
9	0.1	0.2	44.35	29	68	4.55	4	5
10	0.1	0.3	35.25	27	50	4.5	4	5
11	0.1	0.4	26.25	19	34	4.6	4	6
12	0.1	0.5	26.05	16	42	4.4	3	5
13	0.2	0.05	179.05	111	389	2	2	2
14	0.2	0.1	87.75	63	159	2	2	2
15	0.2	0.2	47.75	32	72	2	2	2
16	0.2	0.3	35.6	27	43	2.5	2	3
17	0.2	0.4	32.6	20	63	2.1	2	3
18	0.2	0.5	30.35	21	42	2	2	2
19	0.3	0.05	175.35	91	258	1	1	1
20	0.3	0.1	96.3	41	175	1	1	1
21	0.3	0.2	57	38	88	1	1	1
22	0.3	0.3	34.65	25	44	1	1	1
23	0.3	0.4	35.95	21	91	1	1	1

24	0.3	0.5	31	19	48	1	1	1
25	0.4	0.05	94.1	86	108	1	1	1
26	0.4	0.1	49.95	47	60	1	1	1
27	0.4	0.2	27.85	26	33	1	1	1
28	0.4	0.3	20.35	19	23	1	1	1
29	0.4	0.4	18.55	16	22	1	1	1
30	0.4	0.5	17.55	16	20	1	1	1
31	0.5	0.05	80.45	74	89	1	1	1
32	0.5	0.1	44.95	40	56	1	1	1
33	0.5	0.2	25.4	22	44	1	1	1
34	0.5	0.3	18.45	17	22	1	1	1
35	0.5	0.4	15.35	14	19	1	1	1
36	0.5	0.5	16	13	28	1	1	1

The following observations can be made from the graphs (Appendix II), Figures alongside and Tables 1.1 and 1.2. They were drawn for a set of threshold values 0.5, 0.4, 0.3, 0.2, 0.1, 0.05 and the set of β values 0.5, 0.4, 0.3, 0.2, 0.1, 0.05 for a population of 1000. Each Th- β pair was run for 20 different instances. The least number of peaks were attained when Threshold value was within 0.3. There was only one peak meaning opinion converged to a single value. The maximum number of peaks was 11 obtained for 0.05 Threshold Value.

The above two points can be used to conclude that if the difference between a pair is not considered as a criteria during interactions, then there might never be an agreeable convergence value . Hence the difference between an interacting pair is an important factor. Figure 1 shows behavior of opinions at Th=0.

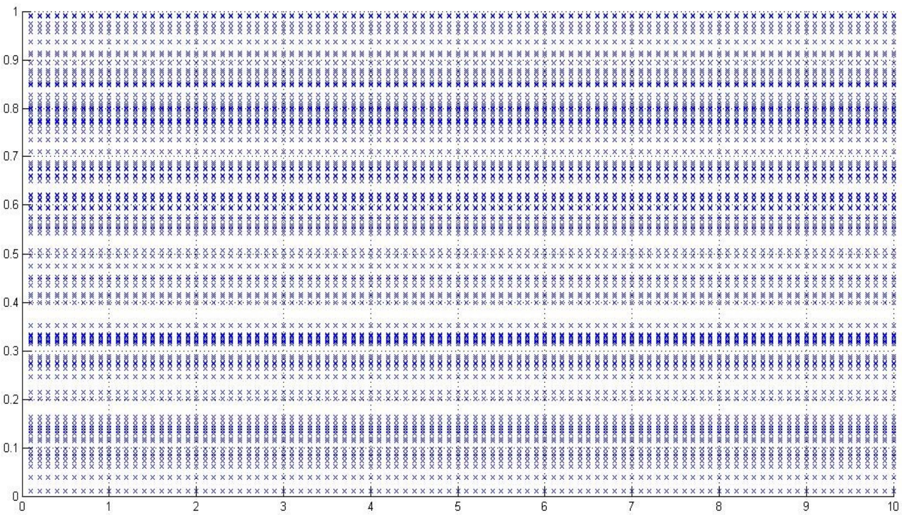

Figure 1: *Convergence of Opinion when there is no Threshold i.e. Th=0,X-axis= Time epochs and Y axis is Opinion*

For a particular threshold, as the Convergence parameter β seems to reduce, the time required to approach convergence increases. So, convergence time can be expected to have inverse proportions with β, the convergence parameter .However, there is no scientific prove for the same apart from a few generalized simulations which follow up. Refer to Table 1.2 , Column of Average Convergence.(Figure 2)

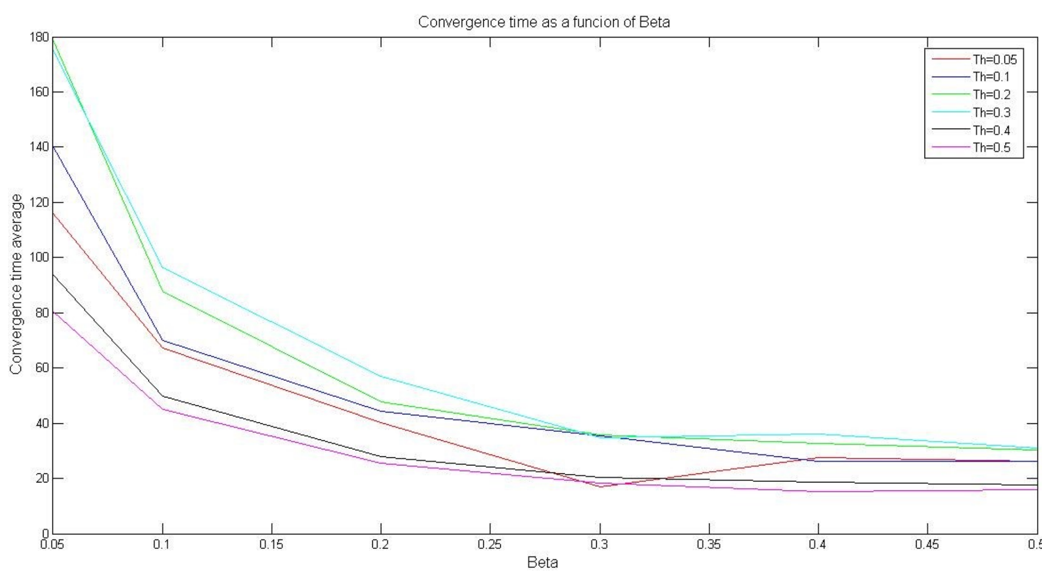

Figure 2: *Convergence time as a function of β when the random vector is changing for every instance*

For different values of β i.e. 0.05, 0.1, 0.2, 0.3, 0.4, 0.5 the average convergence time has been plotted.

For any value of Convergence parameter β, as Threshold value reduces, the population shows clustering patterns may form cavity like conditions within, depending on its value. (Figure 3 and figure 4)

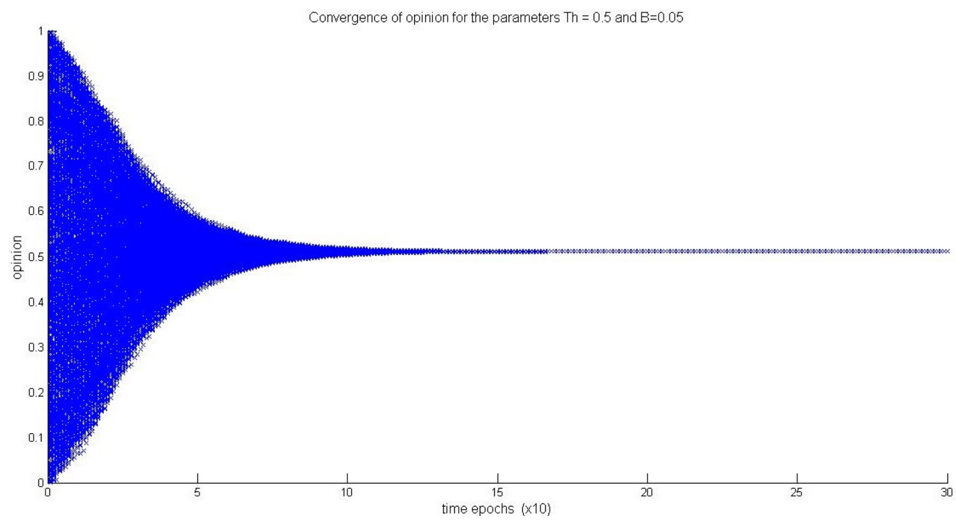

Figure 3: *Convergence of opinion when Th=0.5, β =0.05, population=1000*

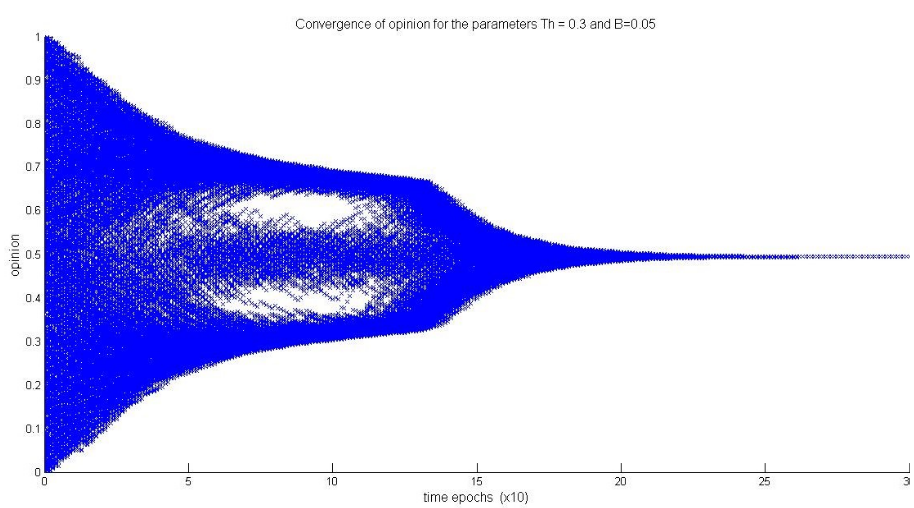

Figure 4: *Th=0.3, β =0.05, population=1000, instance=20*

The cavity implies that the opinions initially form clusters but eventually the clusters converge at a single opinion.

Each converged opinion refers to a peak. The terms peaks and opinion clusters or converged opinions are used interchangeably.

As the Threshold value drops below 0.3, the opinions converge to multiple opinions instead of a single opinion. Refer to Table 1.1 and Figure5 and Figure 6.

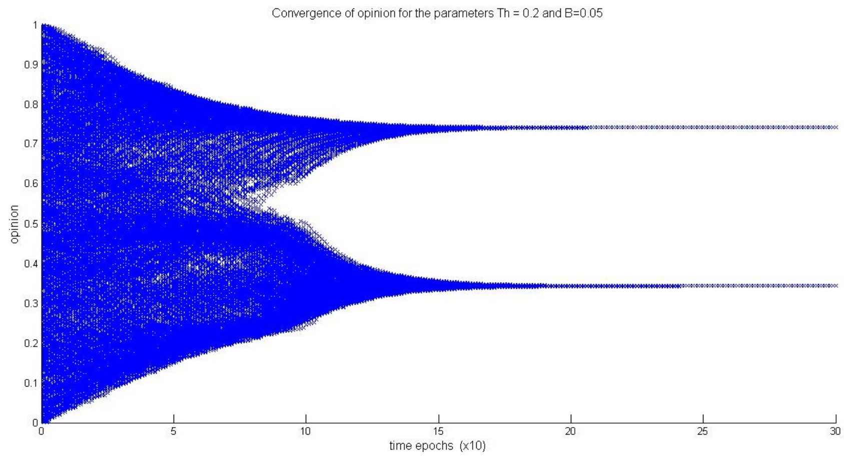

Figure 5: *Convergence of Opinion for Th =0.2, β =0.05, population=1000*

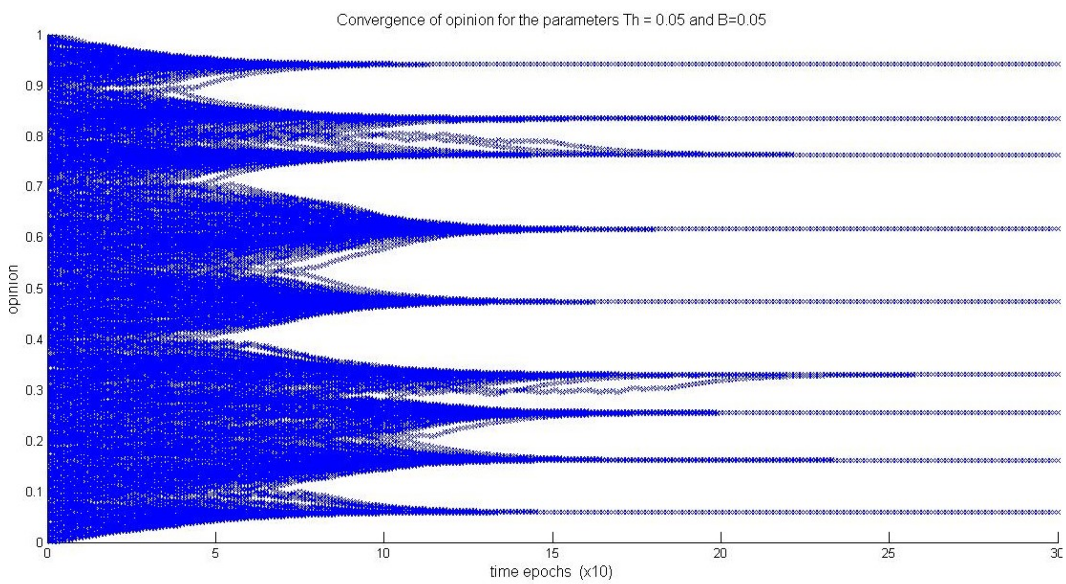

Figure 6: *Convergence of opinion for Th=0.05, β =0.05, population=1000*

These are the conclusions that I have drawn after the simulations. However these are not enough to conclude as each set of data was run for a different set of random variables.

So the simulations were run for the same vector on various Th and β values. β was ranged over a finer set ranging from 0.05 to 05 with a step of 0.01.

The first set of simulations was done to check the convergence time pattern using random pairing schemes. Box plots were done to check the distribution of convergence time for each threshold value. As the threshold value increased, boxplot showed smoother decaying pattern with respect to the β value. Please refer to Appendix II . Figure 7 shows the convergence time versus β plot of a random pairing scheme taken over 20 instances with the same initial opinion at Th =0.5.

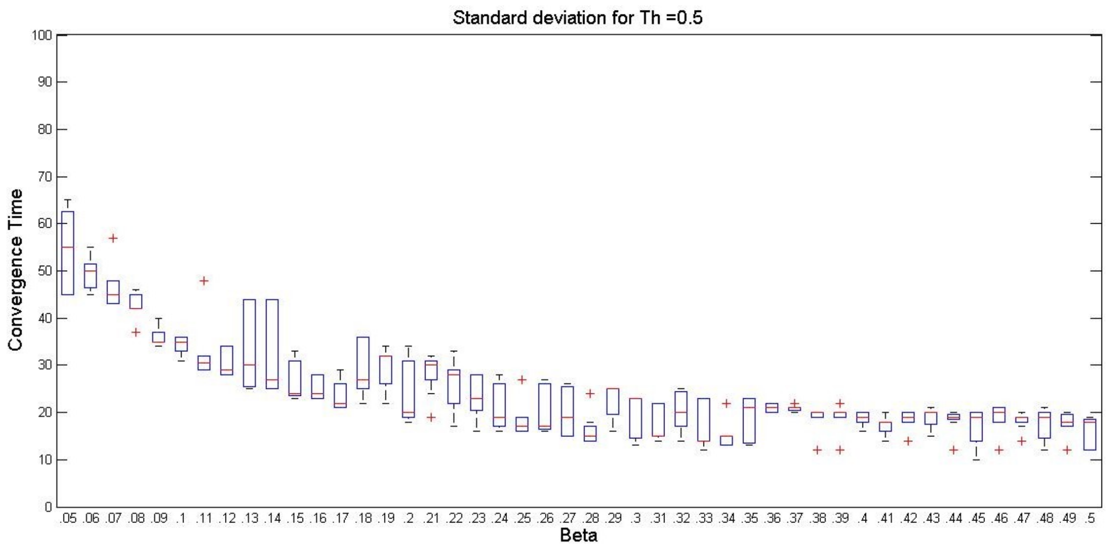

Figure 7: *Box Plot of Convergence Time for Th=0.5*

The convergence Time for deterministic pairing scheme was also simulated and found. It too followed a decaying pattern.

In our Deterministic Pairing scheme, we have paired the two most distant elements having their distance of separation within the threshold.

Figure 8 shows the deterministic pairing of the same random initial opinion vector across 20 instances at Th=0.5. All the 20 instances overlap one over the other because it is the same random vector.

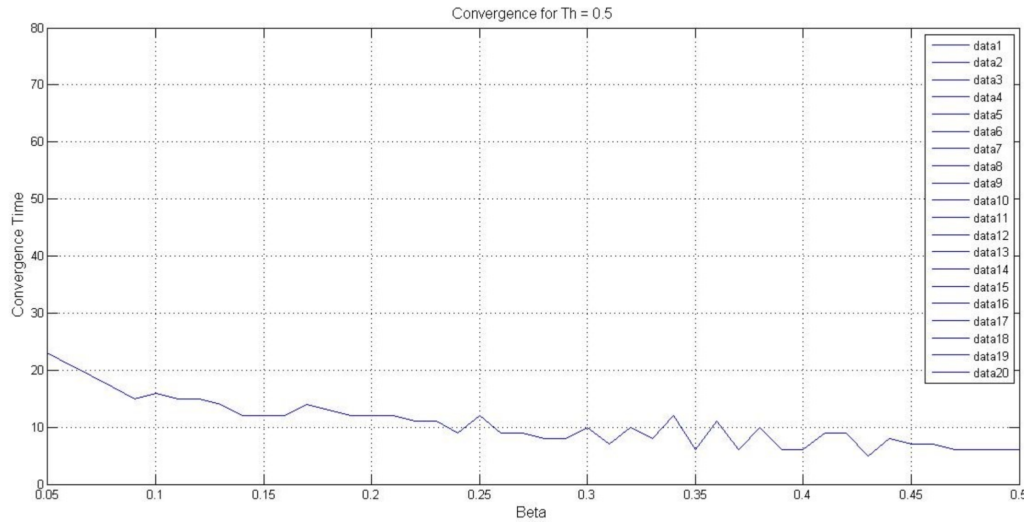

Figure 8 : *Convergence Vs Beta for Th=0.5*

An interesting observation across all these graphs of Convergence time Versus Beta lead to the conclusion that as the Th value increases, the convergence time vs Beta plot smoothens out. Figure 9 shows the Convergence time Versus Beta of Th=0.2. Figure 8 and Figure 9 can be compared to validate the argument. Further validation can be checked by checking the plots from Appendix II.

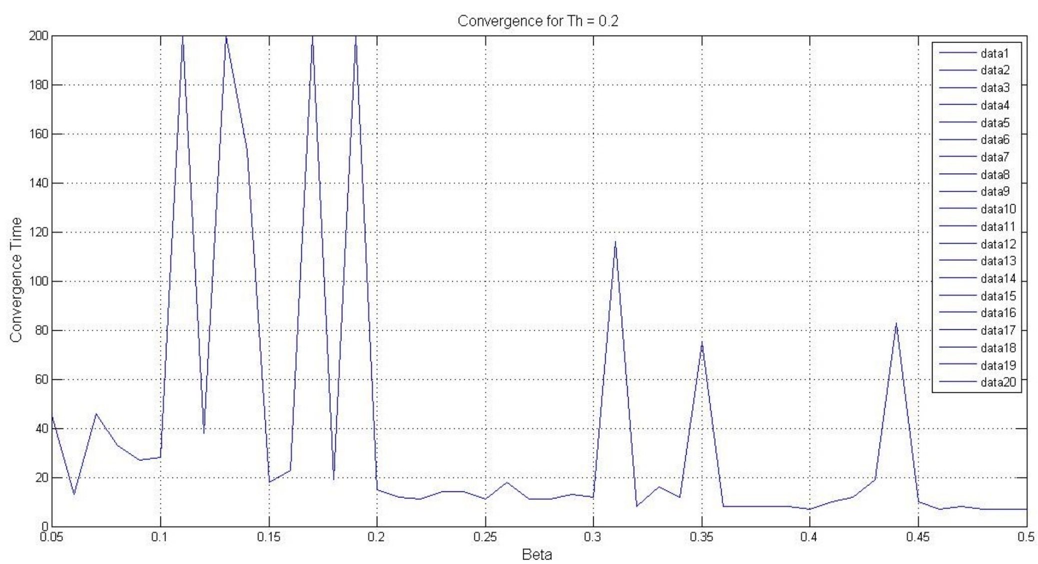

Figure 9: *Convergence time Vs Beta for Th=0.2*

Following up this observation along with the observation that lower is the Threshold value, higher is the number of opinion cluster or peak, as we call each opinion cluster in this report.

Threshold greater than or equal to 0.3 gives a single opinion cluster. This led us to finding out how the peak and convergence time are related.

Figure 10 gives a picture of the peak and convergence time relations at Th= 0.05.

The lower the number of peaks, the more is the time required for it to converge. At this point of time, it must be noted the convergence time is not to be mapped to the real and physical time. For reasons of simplicity convergence time refers to the iteration number during the simulation at which the values lie with a particular convergence criteria. During my simulations, I have considered convergence criteria of 0.0001. That is if the distance between the opinions is less than 0.0001, they are being considered to have converged.

Now, moving back to the original discussion, of Peak with convergence time, we see that for a lower number of peaks, the convergence time is higher. To understand this, say the convergence is to a single peak or opinion cluster. Thus for all the particles irrespective of their distances within the threshold, are bound to converge to the same opinion. Thus the farthest points within the threshold will require more time to converge than the ones nearest to it.

However if there are more number of opinion clusters or peaks, even the opinion within threshold at the boundary will have some opinion cluster being formed near it rather than the other opinion cluster. Thus, it would require lesser time to converge to the opinion.

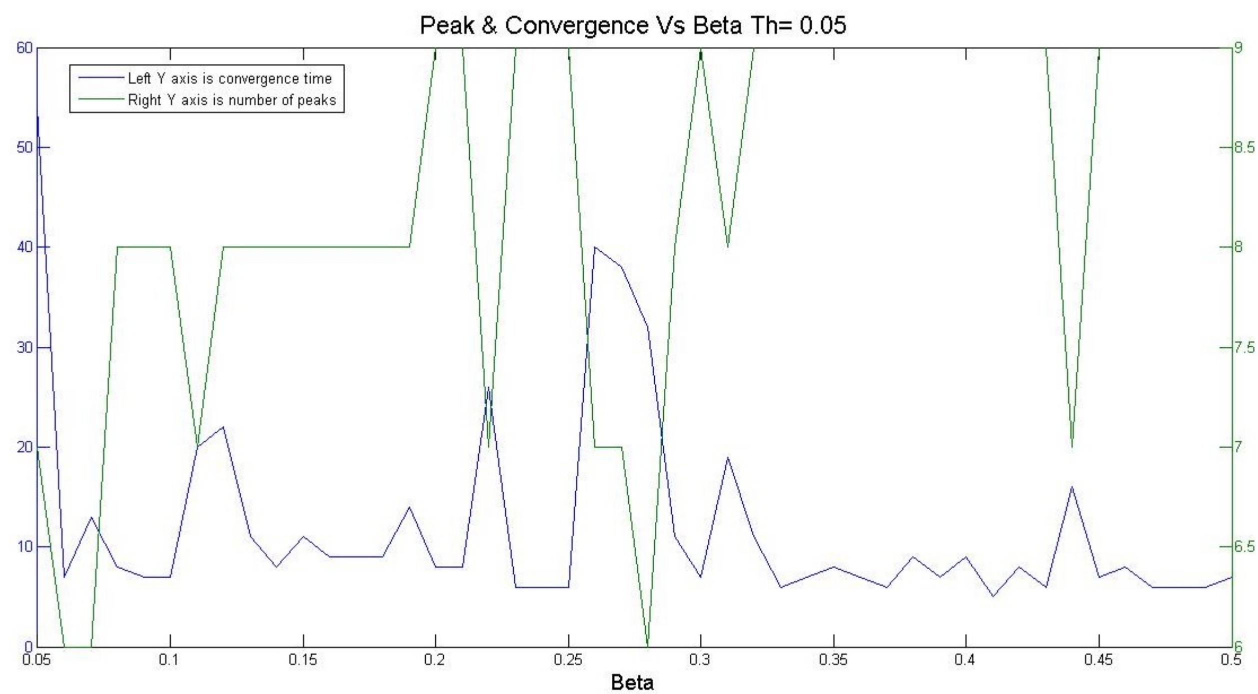

Figure 10: *Plot of Peak and Convergence Time Vs Beta at Th=0.05*

The number of opinion clusters or peaks could be visualized by a plot of peak versus convergence time as on figure 11. However it has been plotted for β ranging from 0.05 to 0.5 at a step of 0.01 and that β has been lost.

The x axis corresponds to the number of peaks or the number of opinion clusters that we have while the y axis corresponds to the convergence time .It is to note that convergence time does not correspond to the real time but to the iteration number at which it satisfies the convergence criteria taken to be 0.0001.

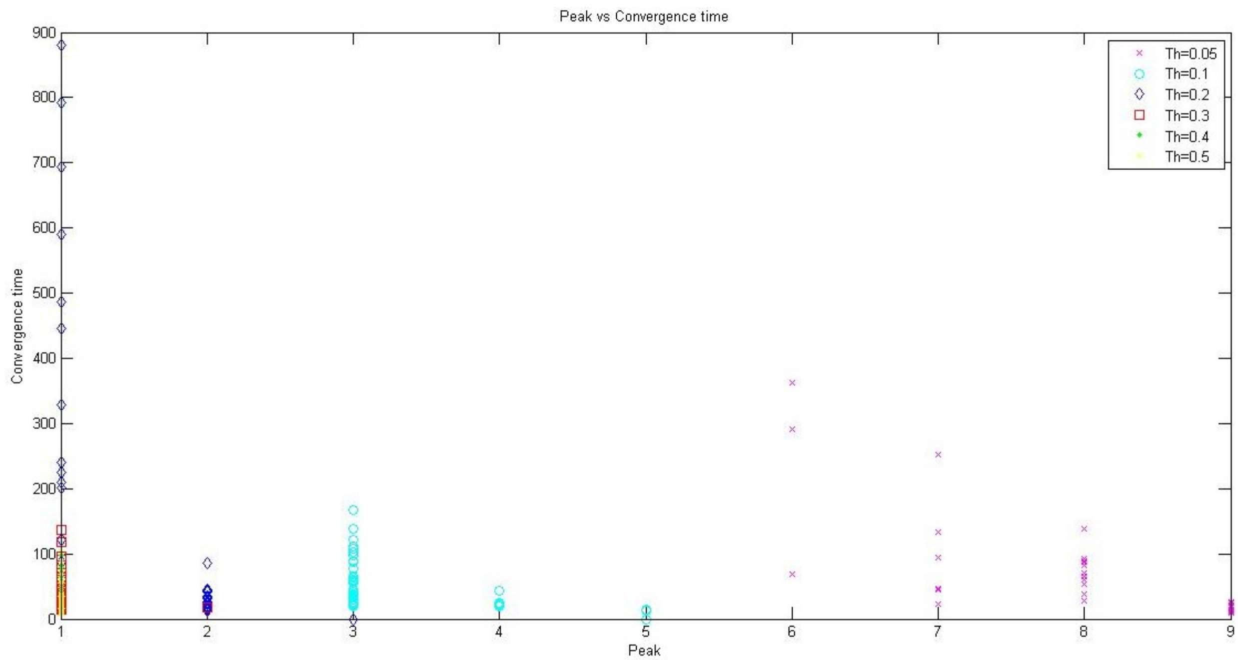

Figure 11: *Plot of Peak Vs Convergence Time*

An important conclusion that can be drawn from this observation is that the more the number of opinion clusters formed, the lesser is the time required for convergence of the opinions. It is clearly visible in Figure 11 that when only one opinion cluster or peak is formed, the convergence time ranges to near about 900 while that for nine opinion clusters. However it is important to note that this is only a general trend. No scientific proofs were looked upon for in support to this trend.

All these simulations were done for a Uniform Distribution of particles over the population size of 100.

An interesting question that arises is what happens if the opinions are already centered at some points? Do they converge at a point or do they stay at their clusters? So to answer the question the simulations were run for Gaussian distributions and also for uniform distribution centered at various points.

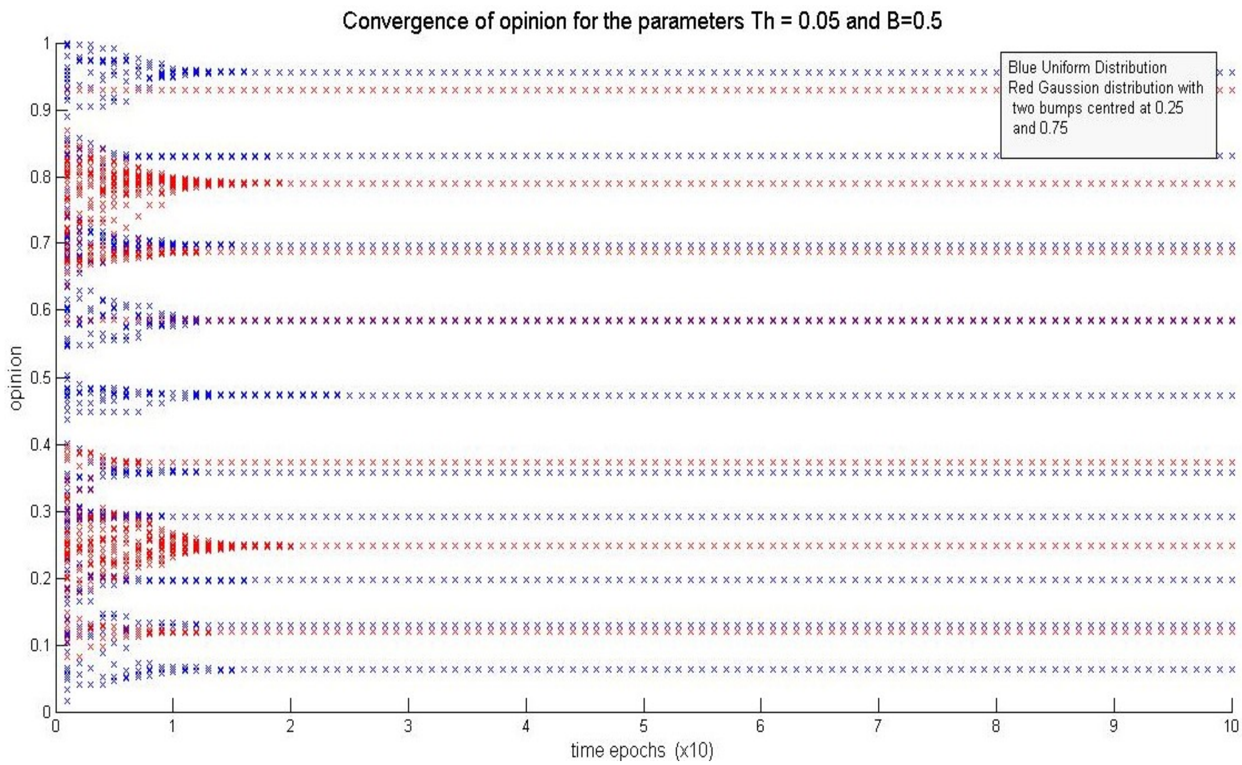

Figure 12: *Uniform distribution and Clustered opinions converging*

Figure 12 shows the comparative figure of a uniform distribution convergence and a Gaussian distribution having two humps whose means are centered at 0.25 and at 0.75.

Figure 13 shows the distribution of the initial versus final opinions.

The blue markers in both Figure 12 and Figure 13 show the uniform distribution of opinions while the red a markers show the Gaussian distribution of opinions having the mean centered at 0.25 and 0.75.

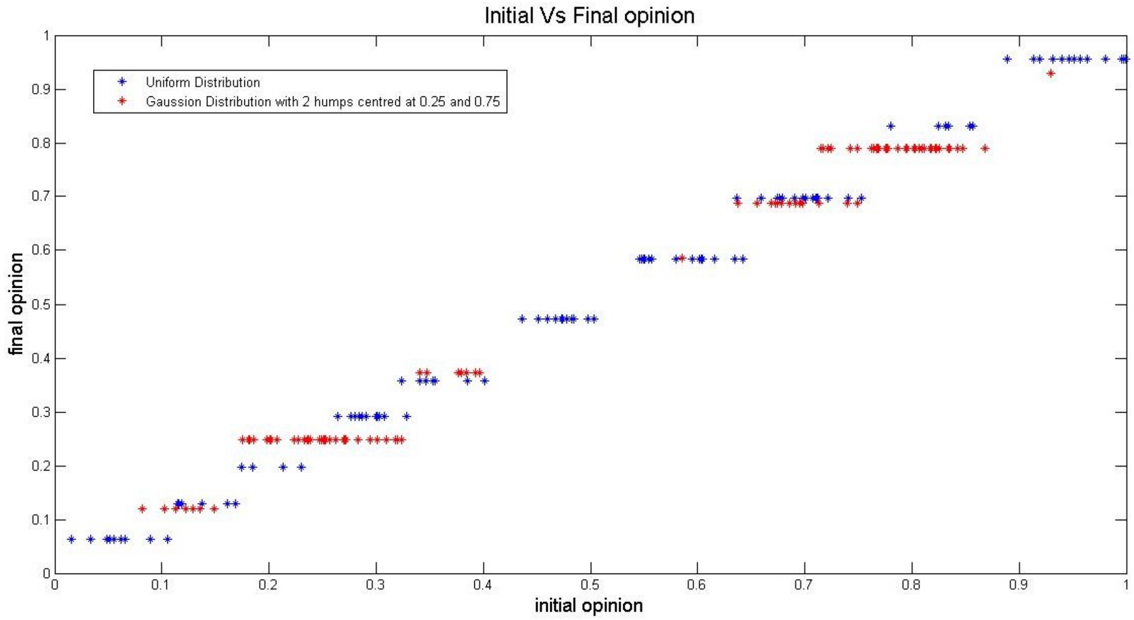

Figure 13: *Initial Vs Final opinion plot of a uniformly distributed opinion vs a double humbed Gaussian distribution at 0.25 and 0.75.*

When seen each case individually for a random pairing scheme, Figure 14 and 15 show the fate of a single Gaussian hump with mean centered at 0.5 with threshold 0.05 and 0.5 respectively.

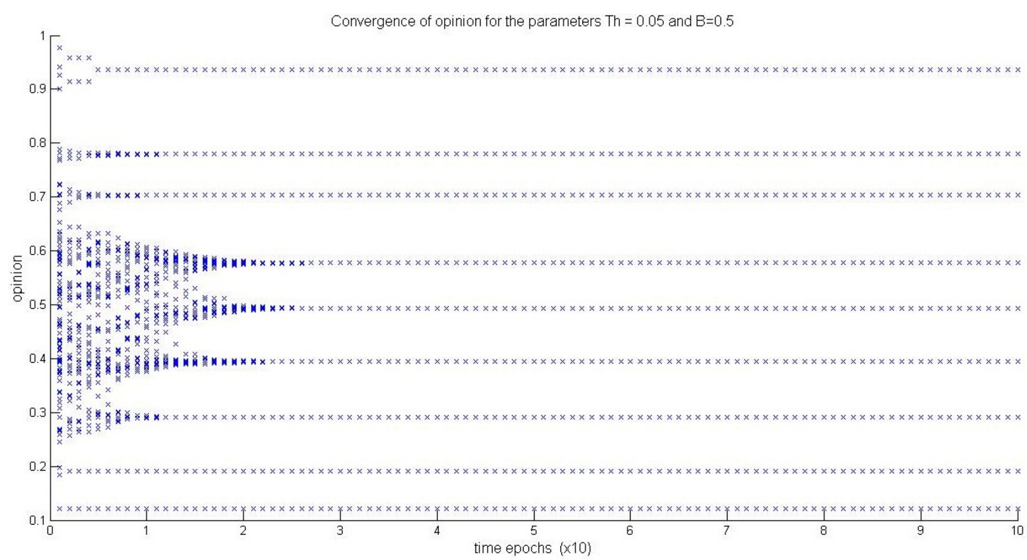

Figure 14: *At Th=0.05, random pairing scheme implemented on a population=100 distributed in a Gaussian fashion with mean Centered at 0.5*

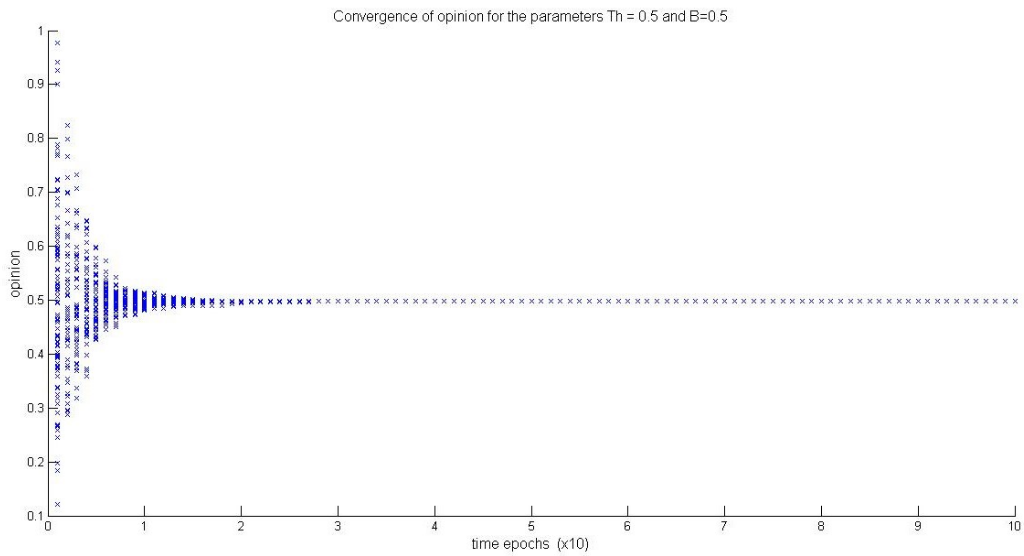

Figure 15: *At Th=0.5, random pairing scheme implemented on a population=100 distributed in a Gaussian fashion with mean centered at 0.5*

Figures 16 and 17 show how a Gaussian distributed population with means centered at 0.25 and 0.75 converge at threshold 0.05 and 0.5 respectively.

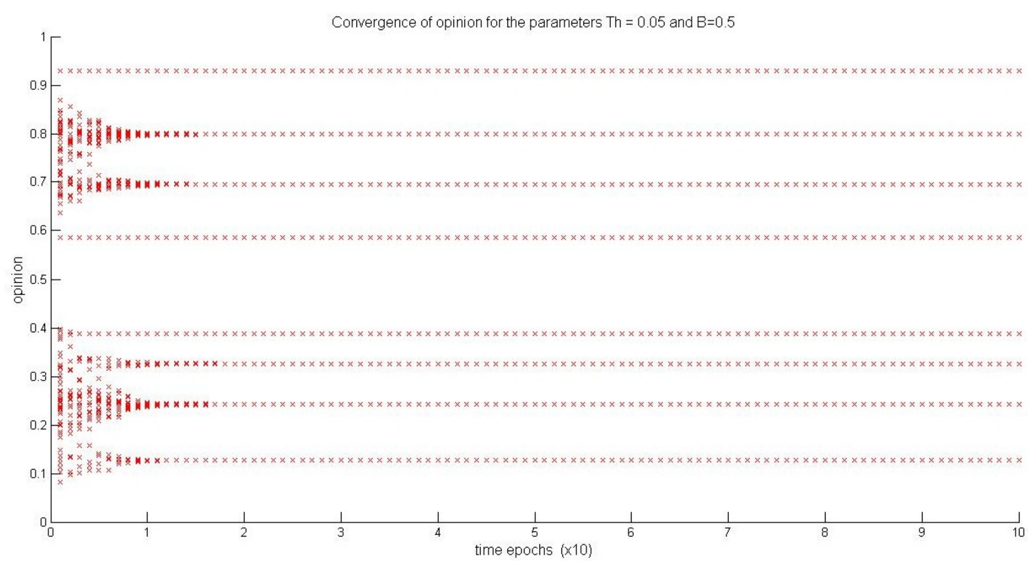

Figure 16: *At Th=0.5, random pairing scheme implemented on a population=100 distributed in a Gaussian fashion with means centered at 0.25 and 0.75*

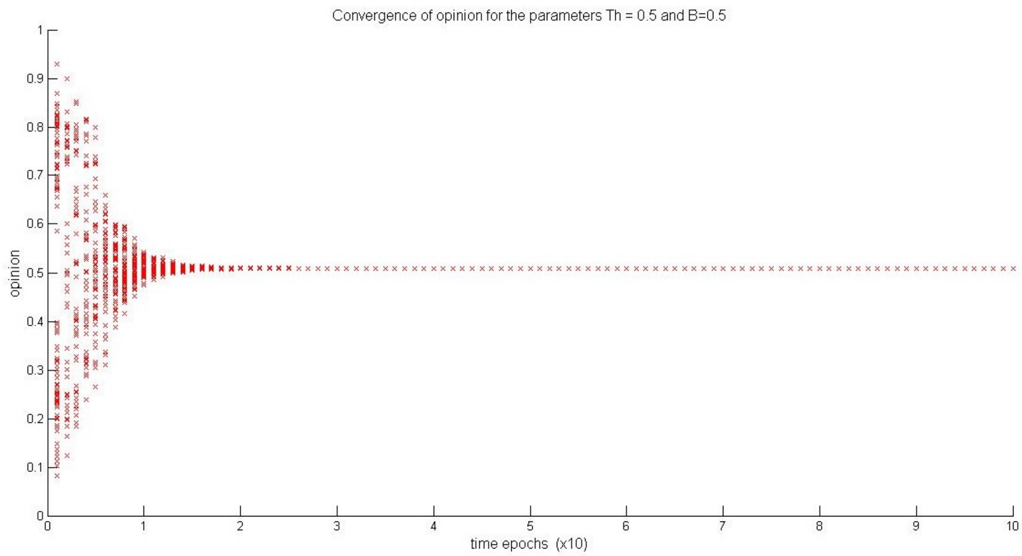

Figure 17: *At Th=0.5, random pairing scheme implemented on a population=100 distributed in a Gaussian fashion with mean Centered at 0.25 and 0.75*

Figure 18 and figure 19 show the convergence of a uniform clustered distribution.

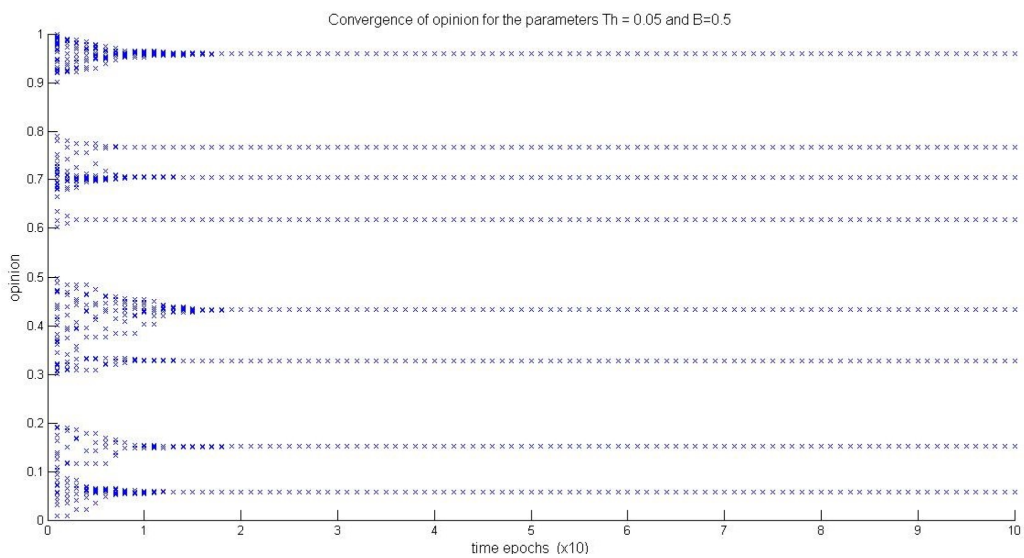

Figure 18: *At Th=0.05, random pairing scheme implemented on a population=100 with uniform clustered opinion*

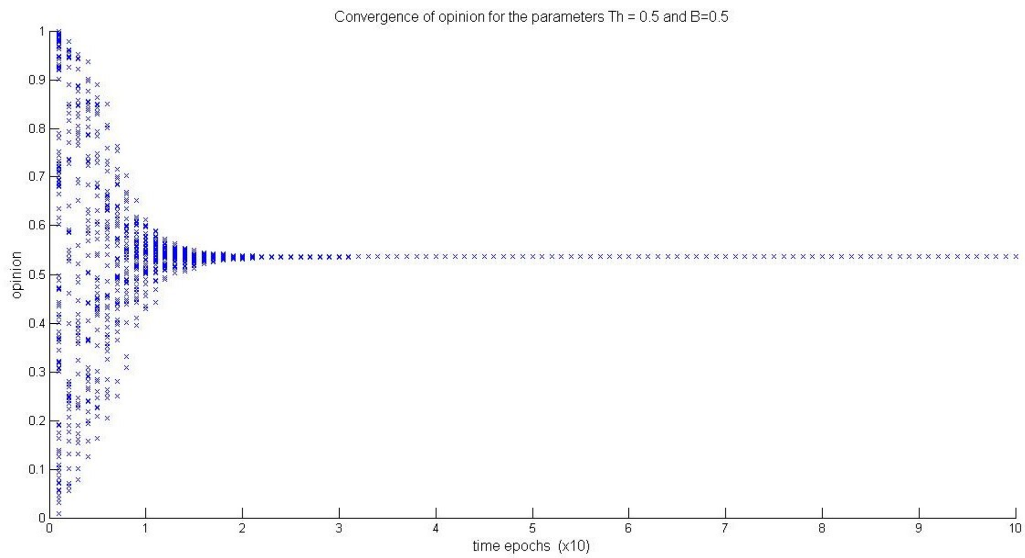

Figure 19: *At Th=0.5, random pairing scheme implemented on a population=100 with uniform clustered opinion*

Figures 20, 21, 22, 23, 24 and 25, show what happens when the deterministic pairing scheme is applied.

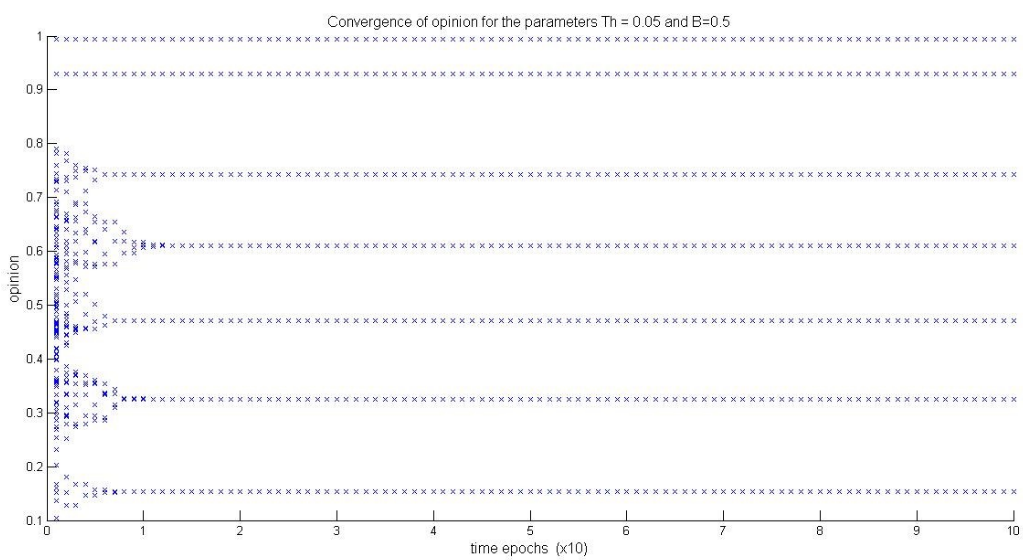

Figure 20: *At Th=0.05, Deterministic pairing scheme implemented on a population=100 distributed in a Gaussian fashion with mean centered at 0.5*

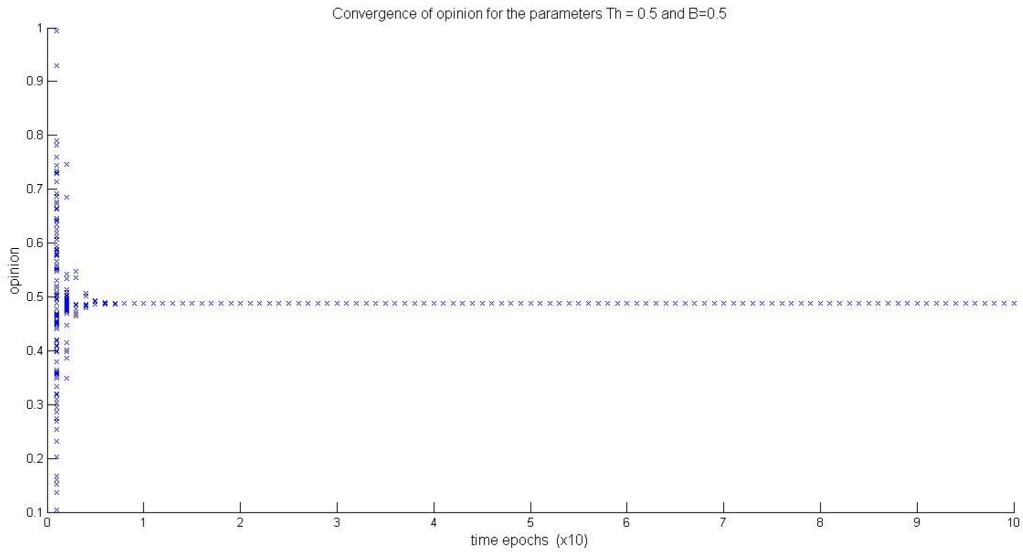

Figure 21: *At Th=0.5, Deterministic pairing scheme implemented on a population=100 distributed in a Gaussian fashion with means centered at 0.5*

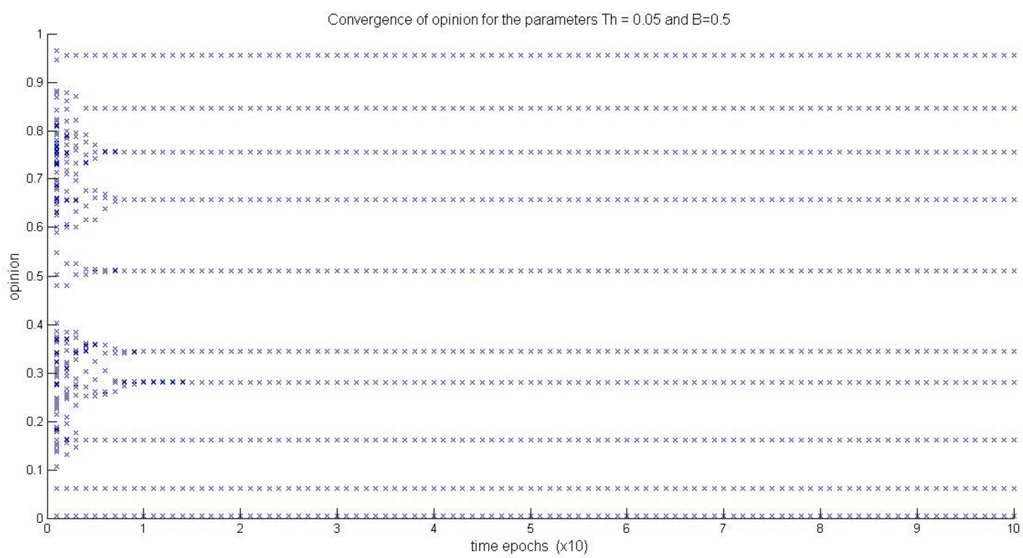

Figure 22: *At Th=0.05, Deterministic pairing scheme implemented on a population=100 distributed in a Gaussian fashion with means centered at 0.25 and 0.75*

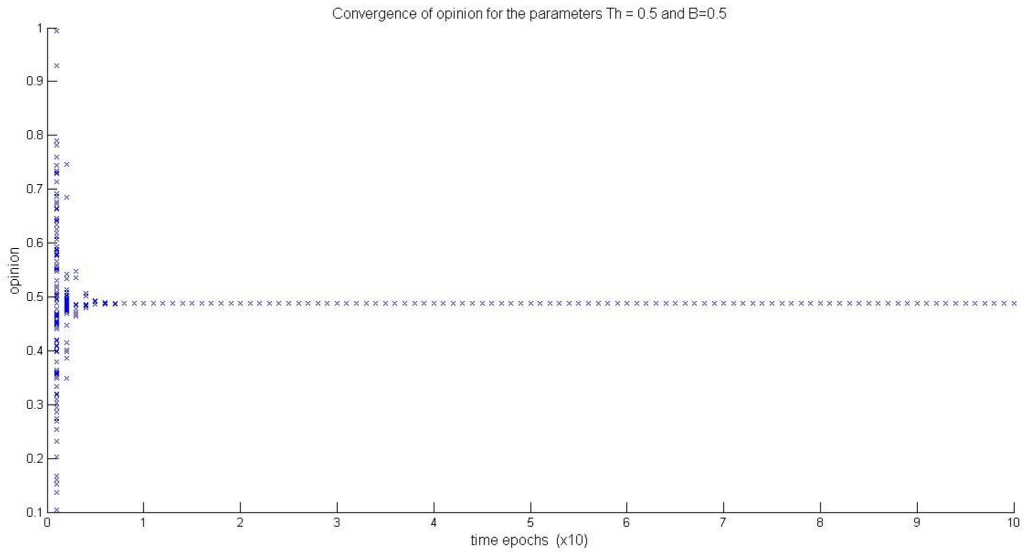

Figure 23: *At Th=0.05, Deterministic pairing scheme implemented on a population=100 distributed in a Gaussian fashion with mean centered at 0.25 and 0.75*

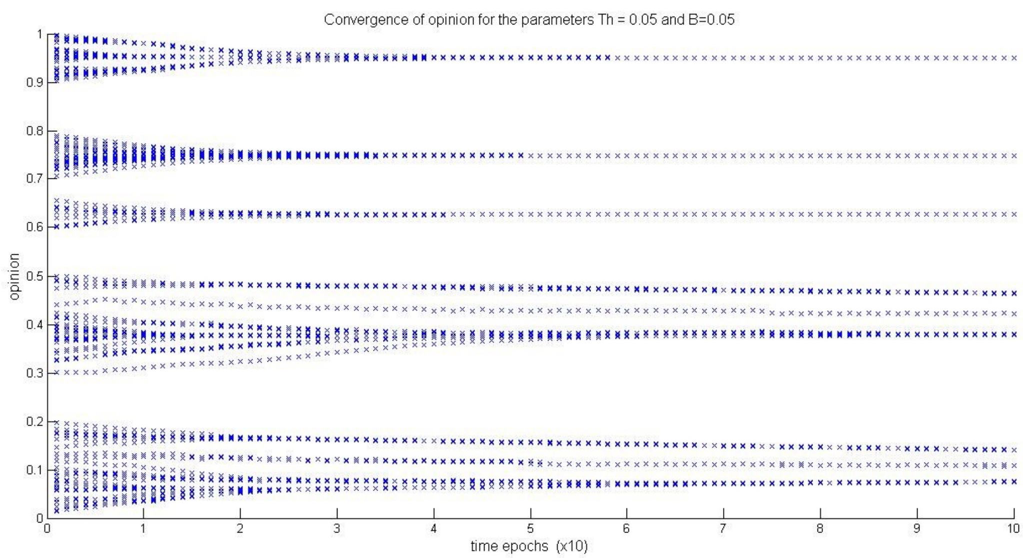

Figure 24: *At Th=0.05, Deterministic pairing scheme implemented on a population=100 with uniform clustered opinion*

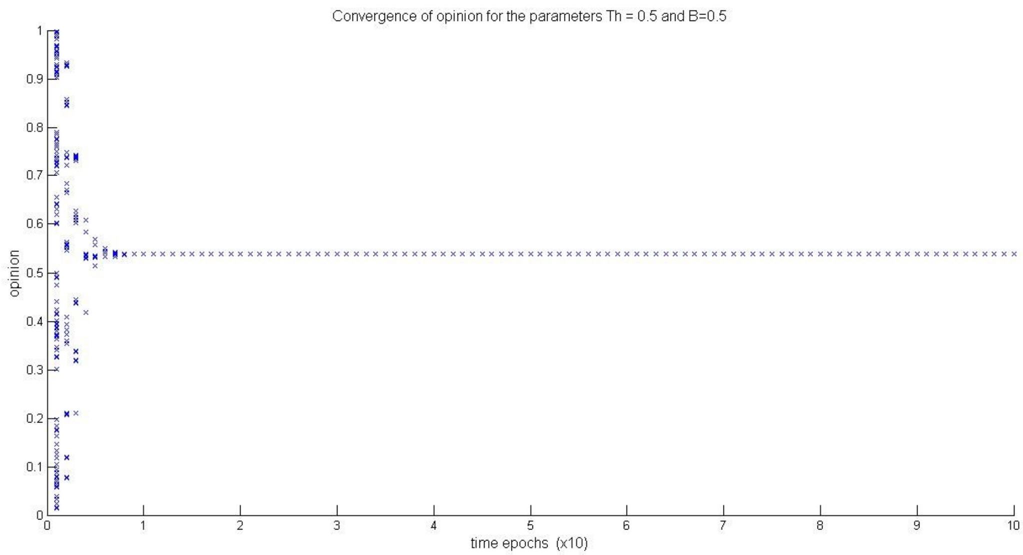

Figure 25: *At Th=0.5, Deterministic pairing scheme implemented on a population=100 with uniform clustered opinion*

We see that even though there is some initial clustering, the system still follows the trend of converging to a single opinion for Th>=0.3 as in 0.5 and to multiple opinion for Th<0.3 as in 0.05.

5. CONCLUSIONS AND RESULTS

The following conclusions could be drawn after all these simulations.

1. As threshold value increases, opinions converge better and the number of opinion clusters decrease.
2. As β increases, the convergence time gradually decreases and seems to have a decaying pattern.
3. The number of peaks formed seems to have inverse relations with convergence time. The more are the opinion clusters, the lesser is the time required for them to converge.
4. The results hold true even if the populations begin with some initial opinion clusters. If Th>= 0.3 they will converge to a single opinion and if Th<0.3 then it will converge to multiple opinion.

6. FUTURE SCOPE OF WORK

Deffuant et al. expands his research to a two-dimensional model and also on social networking site. In this model, the particles are only allowed to interact with their neighbors on the north, east, south and west only. The way they interact remains the same. There are a lot more proposed models on opinion dynamics. There is a paper by Hegselmann et al[3] which uses the generalized means that encompass arithmetic means, geometric means, random means and power means method to understand how opinions evolve over time.

Human opinion dynamics can be used for engineering applications such as solving optimization problems. Kaur et al.[4] had proposed a framework where the system has four main elements which are (a) social structure, (b) opinion space, (c) social influence and (d) updating rule. They have showed that this human opinion dynamics approach based optimization technique performs better than some of the existing swarm based algorithms.

Instead of a simulation based study, one can also perform theoretical analysis on the convergence of the opinions. Degroot had established the convergence criteria for such system in [5] where the system evolves using the following update rule:

$$F^{(n)} = P\, F^{(n-1)},$$

where P is transition matrix and $F^{(n)}$ is the opinion vector at discrete time n.

There are many more models that have been proposed. A study of all these models will increase the depth of understanding and enable its application on real systems and scenarios.

7. REFERENCES

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