Rayleigh-Taylor Instability in anisotropic binary Bose-Einstein Condensates

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Outline of the talk

Bose-Einstein Condensation

- Macroscopic occupation of bosonic particles in the ground state of the system
- Gas of bosonic particles is cooled below a critical temperature T_c condenses into a Bose-Einstein Condensate[1]
- Criteria for condensation when

$$\varpi = n\left(\frac{2\pi\hbar^2}{mKT}\right)^{3/2} \sim 1,$$

  where, ϖ is the phase space density.
- De Broglie wavelength λ_{dB} is comparable to the distance between the particles–wave packets start to overlap –particles become indistinguishable.

[1] Anderson et al., *Science* **269**,(1995), Davies et al. , *Phys. Rev. Lett* **75**, (1995), Ketterle et al. , *Rev. Mod. Phys.* **74**, (2002)

Bose-Einstein Condensation

- At $T = 0$K, the system is fully Bose condensed and can be described by a macroscopic wavefunction

$$\psi(\mathbf{r}) = \sqrt{N}\phi(\mathbf{r}),$$

where, $\phi(\mathbf{r})$ is the single-particle wavefunction. [2]

- To describe both static and time dependent phenomenon in interacting dilute ultracold atomic gases we use Gross-Pitaevskii Equation.

[2] C. Pethick & H. Smith, *Bose-Einstein Condensation in Dilute Gases* (2008)

Gross-Pitaevskii Equation(GPE)

$$i\hbar\frac{\partial\psi}{\partial t} = \Big[-\frac{\hbar^2}{2m}\nabla^2 + V_{trap}(\mathbf{r}) + gN|\psi|^2 \Big]\psi,$$

- $\psi \equiv \psi(\mathbf{r}, t)$: condensate wave function
- $g = \frac{4\pi\hbar^2 a}{m}$
- a: atomic scattering length > 0:repulsive
- N: Number of atoms in the condensate [3]

$$V_{trap} = \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

[3] C. Pethick & H. Smith, *Bose-Einstein Condensation in Dilute Gases* (2008)

Gross-Pitaevskii Equation(GPE)

- GPE is restricted to weakly interacting regime, $n|a|^3 << 1$
- GPE accommodates only isotropic interaction i.e. only s-wave scattering length is sufficient to obtain an accurate description. [4]
- A set of coupled GPE is used to describe two-species Bose-Einstein Condensate.

[4] Dalfovo et al. , *Rev. Mod. Phys.* **71**, (1999)

Coupled Gross-Pitaevskii Equation

$$i\hbar\frac{\partial\psi_1}{\partial t} = \left[-\frac{\hbar^2}{2m_1}\nabla^2 + V_1(\mathbf{r}) + U_{11}N_1|\psi_1|^2 + U_{12}N_2|\psi_2|^2\right]\psi_1,$$
$$i\hbar\frac{\partial\psi_2}{\partial t} = \left[-\frac{\hbar^2}{2m_2}\nabla^2 + V_2(\mathbf{r}) + U_{22}N_2|\psi_2|^2 + U_{12}N_1|\psi_1|^2\right]\psi_2.$$

- $\psi_1 \equiv \psi_1(\mathbf{r}, t)$: condensate wave function for species 1
  $\psi_2 \equiv \psi_2(\mathbf{r}, t)$: condensate wave function for species 2
- $U_{ii} = 4\pi\hbar^2 a_{ii}/m_i$: intraspecies interaction
- $U_{ij} = 2\pi\hbar^2 a_{ij}/m_{ij}$: interspecies interaction
  $m_{ij} = \frac{m_i m_j}{(m_i+m_j)}$ [5]

[5] C. Pethick & H. Smith, *Bose-Einstein Condensation in Dilute Gases* (2008)

Phase Separation

- Miscible Regime

$$U_{11}U_{22} - (U_{12})^2 > 0$$

Coexistence of both the species in some regions of space – partially overlapping wave function

- Immiscible Regime

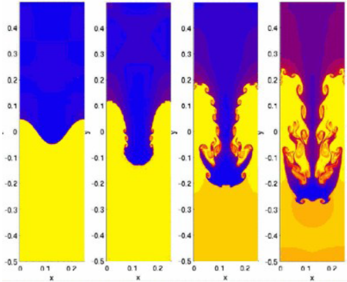
$$U_{11}U_{22} - (U_{12})^2 < 0$$

No coexistence of species in any region of space – separated wavefunction [6]

[6] Ho et al. , *Phys. Rev. Lett.* **77**, (1996)

Rayleigh-Taylor Instability(RTI)

- Instability of an interface when a lighter fluid supports a heavier one in a gravitational field
- Can also occur when a lighter fluid pushes a heavier one
- Leads to turbulent mixing of the two fluids as the perturbations at the interface grow exponentially [7]



Courtesy: en.wikipedia.org

[7] P. G. Drazin & W. H. Reid, *Hydrodynamic Stability* (2004)

RTI in binary Bose-Einstein Condensates

- To initiate RTI[8] in TBEC, we consider harmonic trapping potential.
- We choose the initial state of TBEC to be in immiscible(phase-separated) domain.
- Species with stronger intraspecies repulsive interaction surrounds the other.
- In analogy to normal fluids, species with stronger intraspecies repulsive interaction may be considered to be the lighter fluid.

[8] Gautam et al. , *Phys. Rev. A* **81**, (2010)

Phase-Separated Pan-Cake Shaped TBEC

The trapping potential is

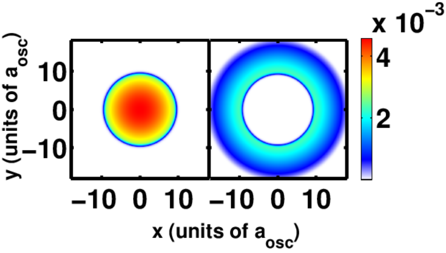
$$V_i(x,y,z) = \frac{m_i\omega^2}{2}(x^2 + \alpha_i^2 y^2 + \lambda_i^2 z^2)$$

- ω : radial trap frequency
- α_i, λ_i: anisotropy parameters
- For pancake shaped trap: $\lambda_i >> 1$
- For simplicity of analysis, we consider, $\alpha_1 = \alpha_2 = \alpha$, $\lambda_1 = \lambda_2 = \lambda$.

Details

In the initial state, at $t = 0$

- We consider a system of ^{85}Rb–^{87}Rb atoms
- $a_{11} = 460a_o$, $a_{22} = 99a_o$, $a_{12} = a_{21} = 214a_o$ [9]
- $N_1 = 5 \times 10^5$ and $N_2 = 10^6$
- $\alpha = 1, \omega_x = \omega_y = 2\pi \times 8Hz$
  $\lambda = 11.25$



[9] Papp et al. , *Phys. Rev. Lett.* **101**, (2008)

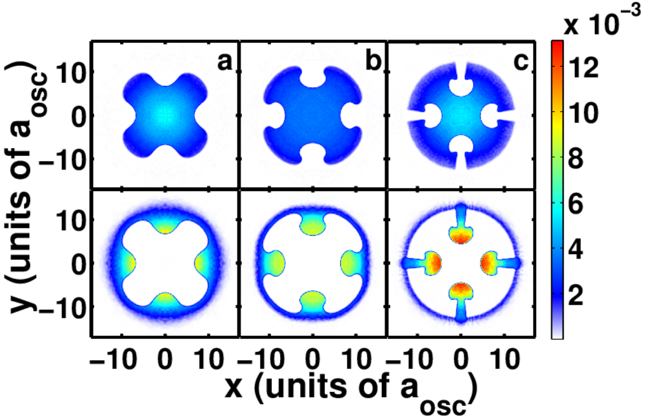
Development of RTI

We initiate RTI in this system by

- Decreasing $a_{11} = 460a_o$ to $a_{11} = 55a_o$ between $t = 0$ ms and $t = 200$ ms
- After $t = 100$ ms, a_{11} is kept fixed. The system is let to evolve freely for another $t = 200$ ms.
- Phase separation condition is maintained throughout the process.
- The outer species tends to come inside the inner species.
- In the process, the circular interface develops instability and grows into mushroom shape pattern.

Development of mushroom pattern

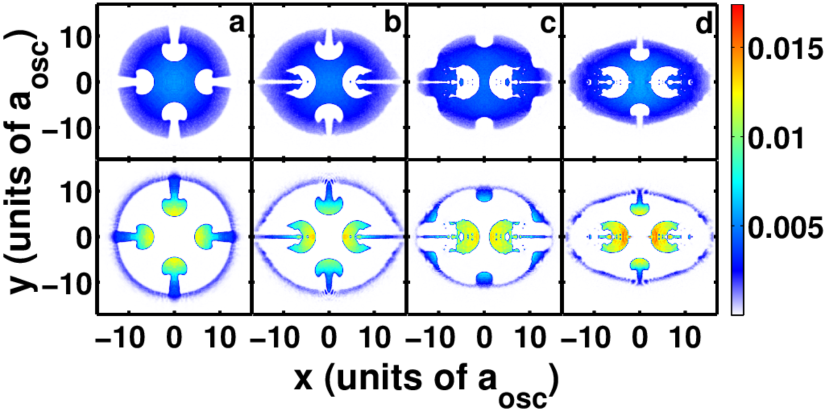
After $t = 358$ ms, $t = 378$ ms, $t = 400$ ms [10]



[10] Sasaki et al. , *Phys. Rev. A* **80**, (2009), Kadokura et al. , *Phys. Rev. A* **85**, (2012)

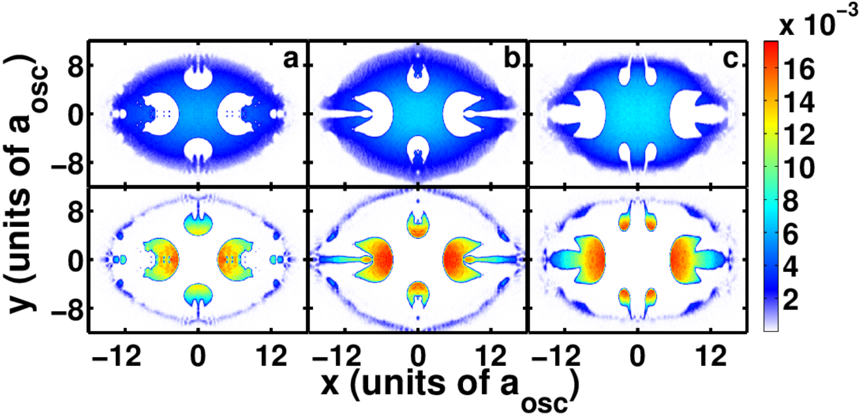
Development of various patterns

On changing the anisotropy of the trap for $\alpha = 1, 1.2, 1.4, 1.6$



Development of various patterns

For $\alpha = 1.8, 2.0, 3.0$

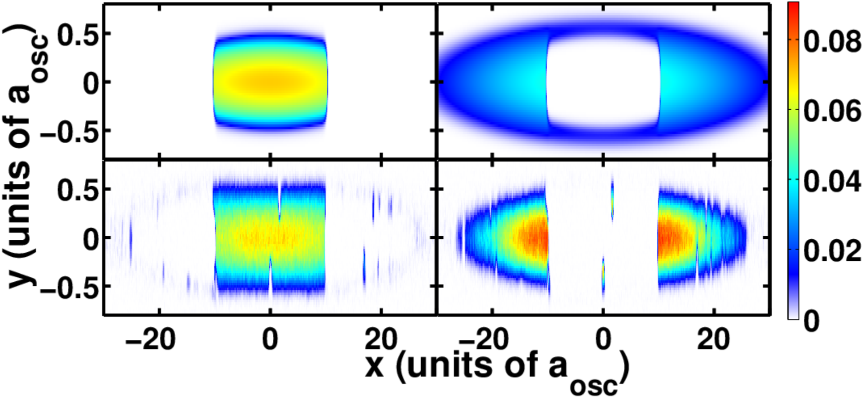


Inhibition of RTI

As α is increased, planar interface is developed

- $a_{11} = 200a_o$, $a_{22} = 99a_o$, $a_{12} = a_{21} = 214a_o$
- $N_1 = 10^5$ and $N_2 = 10^5$
- $\alpha = 50, \omega_x = 2\pi \times 8Hz$
  $\lambda = 100$
- Decreasing $a_{11} = 200a_o$ to $a_{11} = 55a_o$
- Phase separation condition is maintained throughout

Inhibition of RTI



Helmholtz equation

- 2-dimensional Helmholtz equation in Cartesian coordinates(x, y) transformed to elliptic cylindrical coordinates(u, ν)

$$x = a \cosh u \cos \nu,$$
$$y = a \sinh u \sin \nu.$$

- Helmholtz equation in elliptic cylindrical coordinates

$$\frac{1}{a^2(\sinh^2 u + \sin^2 \nu)} \left(\frac{\partial^2 F}{\partial u^2} + \frac{\partial^2 F}{\partial \nu^2} \right) + k^2 F = 0,$$

where, F is the solution of the form $F = U(u)V(\nu)$

Helmholtz Equation

- U's and V's satisfy the Mathieu Equations

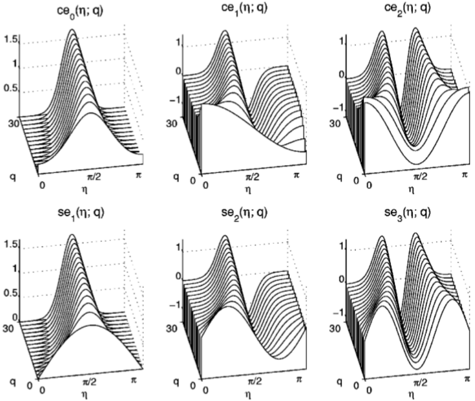
$$\frac{d^2U}{du^2} - \left[\mathcal{A} - 2q\cosh 2u\right] U = 0,$$
$$\frac{d^2V}{d\nu^2} + \left[\mathcal{A} - 2q\cos 2\nu\right] V = 0.$$

where,

$$\mathcal{A} \equiv A + \frac{c^2}{2},$$
$$q \equiv \frac{c^2}{4} \equiv \frac{a^2k^2}{4}.$$

Angular Mathieu Function

Plots of $ce_m(\nu, q)$ and $se_{m+1}(\nu, q)$ on (ν, q) plane. [11]



[11] Gutierrez-Vega et al. , *Am. J. Phys.* **71**, (2003)

Equation of ellipse

$$\frac{x^2}{\beta^2} + \frac{y^2}{\gamma^2} = 1,$$
$$e = \sqrt{1 - \frac{\gamma^2}{\beta^2}}.$$

Again,

$$\beta \sim \frac{1}{\omega_x},$$
$$\gamma \sim \frac{1}{\omega_y}.$$

Therefore, $\frac{\beta}{\gamma} \backsim \alpha$.

The Mathieu equations become

$$\frac{d^2U}{du^2}-\left[A+\left(\frac{k^2}{2}\right)\beta^2\left(1-\frac{1}{\alpha^2}\right)-\left(\frac{k^2}{2}\right)\beta^2\left(1-\frac{1}{\alpha^2}\right)\cosh 2u\right]U=0,$$

$$\frac{d^2V}{d\nu^2}+\left[A+\left(\frac{k^2}{2}\right)\beta^2\left(1-\frac{1}{\alpha^2}\right)-\left(\frac{k^2}{2}\right)\beta^2\left(1-\frac{1}{\alpha^2}\right)\cos 2\nu\right]V=0.$$

THANK YOU