Mode bifurcation in the Rayleigh-Taylor instability of binary condensates

Arko Roy[1], S. Gautam[2], and D. Angom[1]

[1]Physical Research Laboratory, Ahmedabad - 380 009, Gujarat, India
[2]Department of Physics, Indian Institute of Science, Bangalore - 560 012, India

Physical Research Laboratory

Introduction

- **Rayleigh-Taylor Instability(RTI)** is an instability of an interface when a **lighter fluid supports a heavier one** in a gravitational field or some external potential.
- It can also occur when a **lighter fluid pushes a heavier one**.
- Occurs due to **unfavourable energy conditions** and as a result, the fluids tend to swap their positions.
- Leads to turbulent mixing of the two fluids as the **perturbations** at the interface **grow exponentially**.

RTI in binary Bose-Einstein condensates

Objectives:

- We examine the generation and subsequent evolution of RTI in anisotropic **two-species Bose-Einstein condensates(TBEC)** in a **pancake-shaped trap**.
- Initiate instability, we **tune intraspecies interaction** between atoms.
- We analytically study **normal modes** of interface using elliptic cylindrical coordinates.
- We find the normal modes undergo **bifurcation** at particular values of anisotropy. and ratio of number of atoms.

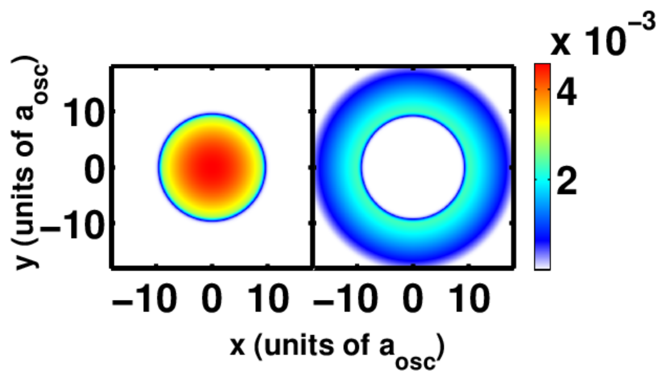
Phase separated pancake-shaped TBECs

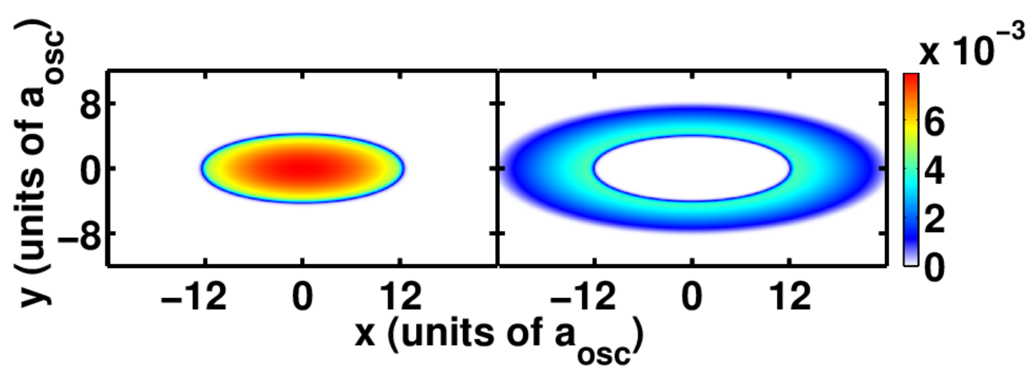
- **2-D Gross-Pitaevskii(GP) equation**(scaled units)

$$\left[-\nabla_\perp^2 + V_k(x,y) + \sum_{j=1}^{2} \mathcal{N}_{kj}|\phi_j(x,y)|^2\right]\phi_k(x,y) = i\frac{\partial \phi_k}{\partial t},$$

$$V(x,y) = \frac{m\omega^2}{2}(x^2 + \alpha^2 y^2)$$

- In the initial state, at $t = 0$
  - We consider a system of ^{85}Rb–^{87}Rb atoms
  - $a_{11} = 460a_B$, $a_{22} = 99a_B$, $a_{12} = a_{21} = 214a_B$





Normal modes of the interface

Helmholtz equation in 2D(scaled units)

$$(\nabla_\perp^2 + \Lambda_k^2)\phi_k = 0$$

$$\Lambda_k^2 = \tilde{\mu}_k - \alpha,\ \tilde{\mu}_k = \mu_k - \sum_{j=1}^{2}\mathcal{N}_{kj}|\phi_j(x,y)|^2$$

- This equation is **valid** only at the **interface** or close to it, where **densities are low**.
- In **elliptic cylindrical coordinates**, Helmholtz equation assumes the form of Mathieu equations

$$\frac{d^2U}{du^2} - \left[\mathcal{A} + \frac{1}{2}\Lambda^2 e^2\alpha\,(1-\cosh 2u)\right]U = 0,$$

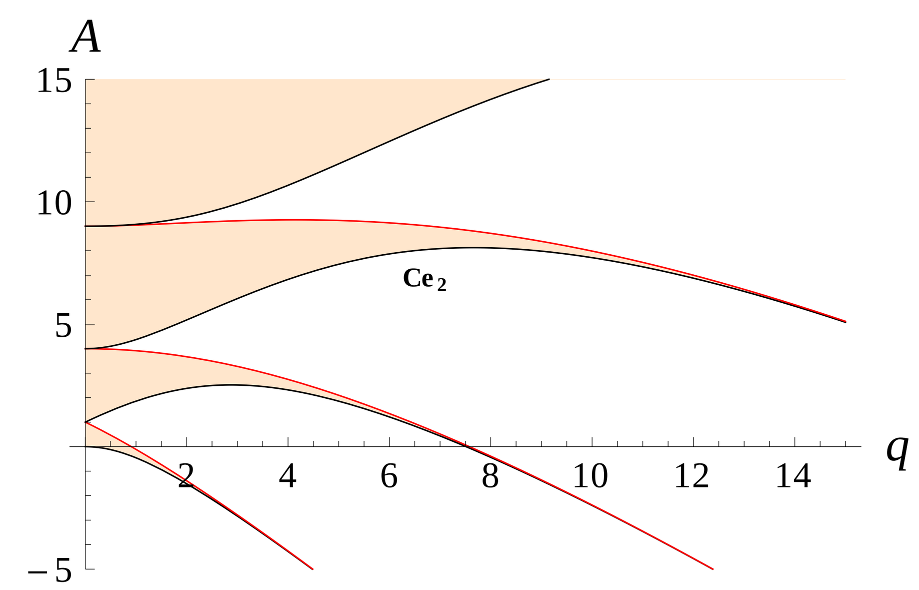
$$\frac{d^2\Theta}{dv^2} + \left[\mathcal{A} + \frac{1}{2}\Lambda^2 e^2\alpha\,(1-\cos 2v)\right]\Theta = 0.$$

$$q = \Lambda^2 e^2\alpha/4,\ e = \sqrt{1 - 1/\alpha^2}$$

- u represents the **ellipse**, v is the **angular coordinate** which varies and lies in the domain $[0, 2\pi)$.

Allowed solutions

- Solutions of the **angular Mathieu equation**, are the $\mathrm{ce}_{n-1}(v,q)$ and $\mathrm{se}_n(v,q)$ functions, cosine and sine elliptic functions, respectively.



- The solution of interest is $\mathrm{ce}_2(v,q)$, it satisfies
  - $\mathrm{ce}_2(v,q) = \mathrm{ce}_2(v+\pi,q)$ and
  - $\mathrm{ce}_2(v,q) = \mathrm{ce}_2(-v,q)$
  - $\mathrm{ce}_2(v,q)$ is maximum at $v = 0$
  - Undergoes a smooth **bifurcation** at **higher** values of q.
- At higher anisotropy parameter α, instead of **four** there must be **six** mushroom shaped inward superfluid flow, which matches with our numerical results.

Growth Rate

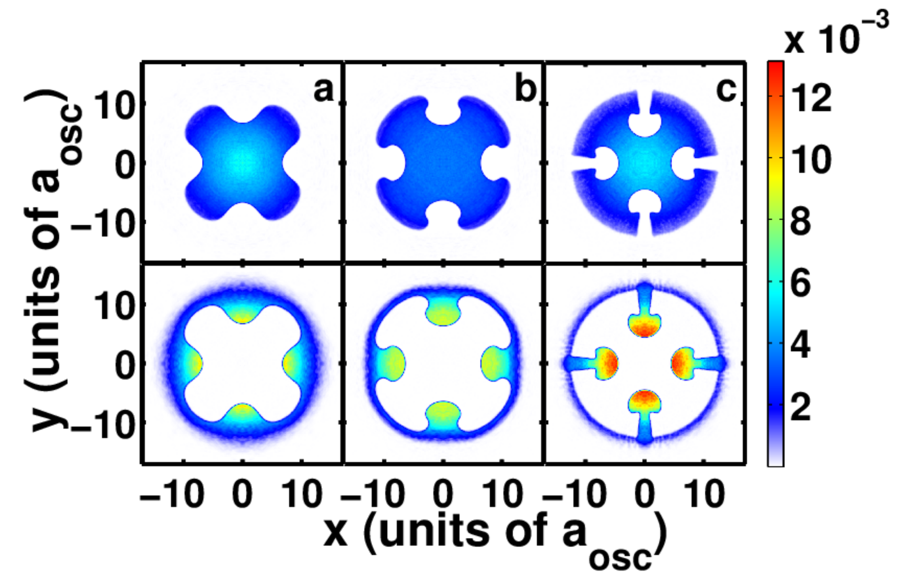
$$s = \pm[\mathcal{A} + 2q\,(1-\cos 2v)]^{\frac{1}{4}}\left[\frac{g(n_1 - n_2)}{n_1 + n_2}\right]^{\frac{1}{2}}$$

- $n_1(n_2)$ refers to density of **outer(inner)** species.
- **RTI** sets in when $n_1 > n_2$.

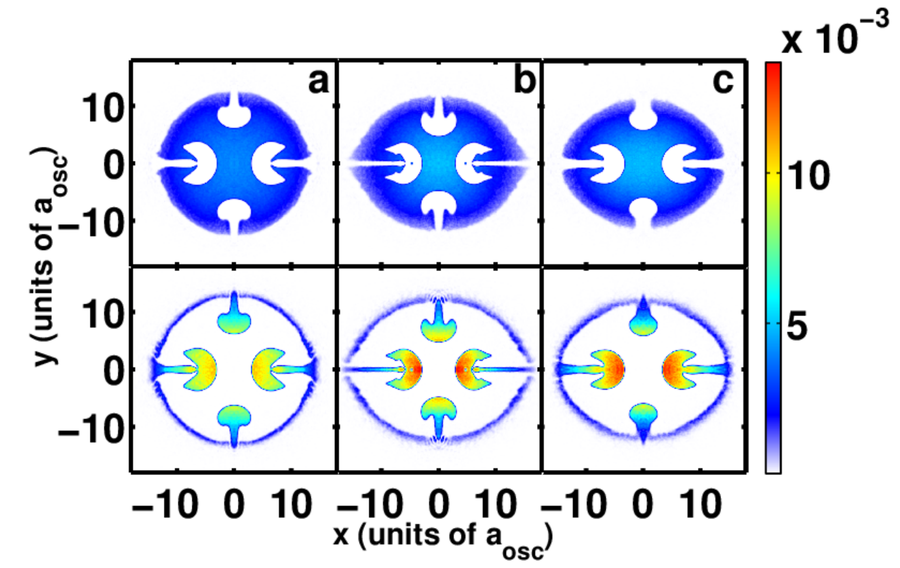
Numerical Results

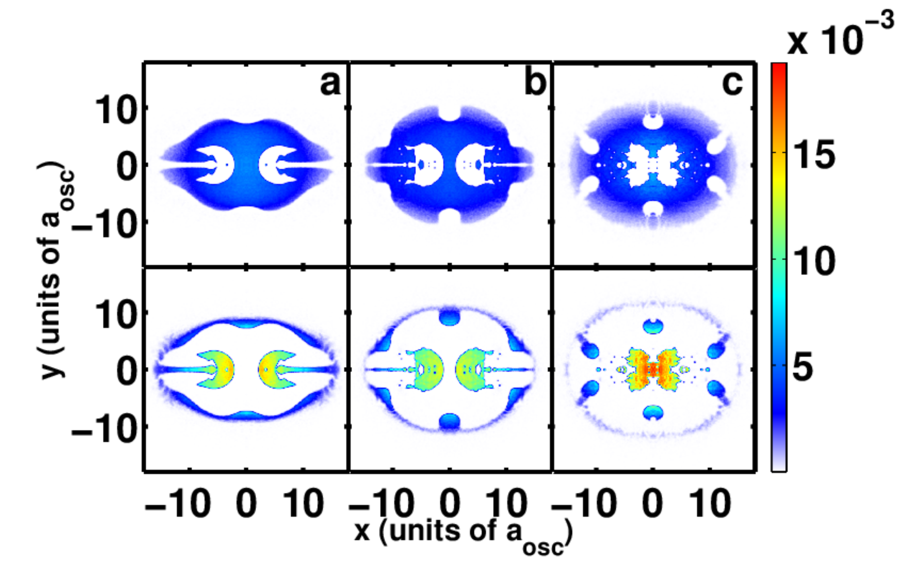
Mode bifurcation and density profiles

For, $\alpha = 1$



For, $\alpha > 1$





Salient Features:

- Density profiles show early stages of RTI.
- Development of non-linear patterns on changing the anisotropy of the trap.
- Bifurcation occurs at particular value of α.

References

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Mail: arkoroy@prl.res.in