On finite temperature effects in Bose-Einstein condensates

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YOUNG PHYSICISTS' MEET (YPM) - 2014

AR, S. Gautam, and D. Angom, *Phys. Rev. A*, **89**, 013617 (2014)
S. Gautam, AR, and S. Mukerjee, *Phys. Rev. A*, **89**, 013612 (2014)

Plan of the talk

Bose-Einstein Condensation

- Macroscopic occupation of non-interacting bosons in the ground state of the system
- A gas of bosonic particles cooled below a critical temperature T_c condenses into an ideal Bose-Einstein condensate (BEC)
- Criteria for condensation

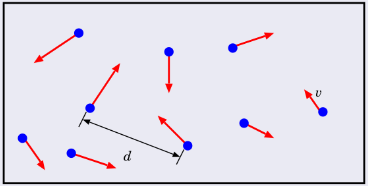
$$\varpi = n\left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2} = 2.612,$$

- De Broglie wavelength λ_{dB} comparable to the distance between the particles–wave packets start to overlap

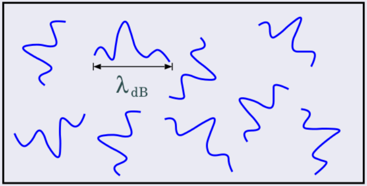
Anderson *et. al*, *Science* **269**, (1995);
Davies *et. al*, *Phys. Rev. Lett.* **75**, (1995);
Ketterle *et. al*, *Rev. Mod. Phys.* **74**, (2002).

Basic Phenomenon

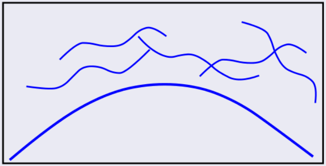
High Temperature (T), Density d^{-3}



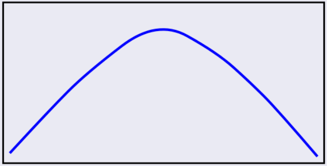
Low Temperature (T), λ_{dB}



Bose-Einstein Condensate, $T = T_{crit}$



Pure Bose Condensate, $T = 0$



P. Muruganandam (Workshop on HPC, PRL Ahmedabad, 2012).

General Criteria for BEC

When interactions are present$\Rightarrow$ Single-particle energy levels are not defined. A reduced *single-particle density operator* is defined

$$\hat{\rho}_1 \equiv \mathrm{Tr}_{2,3,\cdots N}\, \hat{\rho}$$

where $\mathrm{Tr}_{2,3,\cdots N}$ $\rightarrow$Trace of $\hat{\rho}$ w.r.t particles $2, 3, \cdots N$

- Define $\hat{\sigma}_1 = N\hat{\rho}_1$
- **Penrose-Onsager condition**:

$$\frac{n_M}{N} = e^{\mathcal{O}(1)}$$

- n_M: largest eigenvalue of $\hat{\sigma}_1$, condensation occurs in corresponding eigenstate
- $e^{\mathcal{O}(1)}$: positive number of the order of unity.

O. Penrose and L. Onsager, *Phys. Rev*, **104**, (1956)

Gross-Pitaevskii equation

- Equation of motion of the condensate wavefunction is given by Gross-Pitaevskii equation (GPE), strictly valid at $T = 0\mathrm{K}$.

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}}(\mathbf{r}) + gN|\psi|^2\right]\psi,$$

  - $\psi \equiv \psi(\mathbf{r}, t)$: condensate wave function
  - $g = \dfrac{4\pi\hbar^2 a}{m}$
  - a: *s-wave* scattering length > 0 : repulsive
  - N: Number of atoms in the condensate

$$V_{\mathrm{trap}} = \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

E. P. Gross, *Il Nuovo Cimento Series 10*, **20**, (1961);

L. P. Pitaevskii, *Soviet Physics JETP-USSR*, **13**, (1961);

C. Pethick & H. Smith, *Bose-Einstein Condensation in Dilute Gases*, (2008)

Why do we study finite temperature effects?

Region of interest :: $0 < T < T_c$

- $T = 0K$ is physically unattainable. Experiments take place at finite temperatures.
- When $T \neq 0$, the condensate co-exists with the *thermal cloud*. Interactions between condensate and non-condensate (thermal) atoms cannot be neglected.

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Modify GPE to incorporate the effects of temperature.

Finite Temperature models

Stationary Case:

- Hartree-Fock-Bogoliubov-Popov approximation
- Modified Popov approximation

AR, S. Gautam, and D. Angom, *Phys. Rev. A*, **89**, (2014);
Blakie *et. al*, *Adv. Phys.*, **57**, (2008);
A. J. Allen, Ph.D. Thesis (2010);
N. P. Proukakis and B. Jackson, *J. Phys. B*, **41**, (2008)

Finite Temperature models

Stationary Case:

- Hartree-Fock-Bogoliubov-Popov approximation
- Modified Popov approximation

Dynamical Case:

- Projected Gross-Pitaevskii equation
- Truncated Wigner approximation
- Self-consistent Gross-Pitaevskii-Boltzmann (ZNG formalism)
- Dissipative Gross-Pitaevskii equation
- Stochastic Gross-Pitaevskii equation (SGPE)

AR, S. Gautam, and D. Angom, *Phys. Rev. A*, **89**, (2014);
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Langevin equation of Brownian motion

- The apparently random movement of a particle in a fluid due to collisions with the molecules of the fluid is termed as **Brownian motion**.
- The equation of motion for a particle of mass m, subjected to the frictional force, is given by Stokes' law $F = -\alpha v$

$$\begin{aligned} m\frac{dv}{dt} &= -\alpha v \\ \Rightarrow v &= v_0 e^{-\gamma t} \end{aligned}$$

- Solution implies that the motion will dissipate with a characteristic time $\tau = 1/\gamma = m/\alpha$
- The average thermal velocity of a particle within a liquid at some temperature T

$$v_t = \sqrt{\frac{kT}{m}}$$

Langevin equation

- To take into account the random thermal background motion, the force acting on the particle is written as a sum of a viscous force proportional to the particle's velocity (Stokes' law), and a noise term $\Gamma(t)$

$$\frac{dv}{dt} = -\gamma v + \Gamma(t)$$

- Noise term (Langevin force), which has Gaussian probability distribution, satisfies the relations

$$\langle\Gamma(t)\rangle = 0, \langle\Gamma(t)\Gamma(t')\rangle = q\delta(t - t'),$$

- The solution of the Langevin equation

$$v(t) = v_0 e^{-\gamma t} + e^{-\gamma t}\int_0^t e^{\gamma t'}\Gamma(t')dt'$$

N. Pottier, *Nonequilibrium Statistical Physics*, (2010)

Fluctuation-dissipation relation

- Velocity correlation between two times in long time limit $(t_1, t_2 \gg \gamma^{-1})$

$$\langle v(t_1)v(t_2)\rangle = \frac{q}{2\gamma}[e^{-\gamma|t_1-t_2|}]$$

- Equipartition relation

$$\frac{1}{2}m\langle[v(t)]^2\rangle = \frac{1}{2}kT$$

- Relation between the strength of the noise and the magnitude of the damping

$$q = \frac{2\gamma kT}{m}$$

This relation must be satisfied if thermal equilibrium is to be reached at long times.

Stochastic GP equation / Langevin equation for BEC

- At **finite temperatures** a BEC can described by Stochastic GP equation (SGPE),

$$i\hbar\frac{\partial\Psi}{\partial t} = (1-i\gamma)\left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}} - \mu + g|\Psi|^2\right]\Psi(\mathbf{x},t)+\eta(\mathbf{x},t)$$

  $\gamma \rightarrow$ Dissipation, $\mu \rightarrow$ Chemical Potential
- The fluctuating noise term satisfies

$$\begin{aligned} \langle\eta(\mathbf{x},t)\rangle &= 0 \\ \langle\eta(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle &= 2\gamma kT\hbar\delta(\mathbf{x}-\mathbf{x}')\delta(t-t') \end{aligned}$$

- The strength of the noise is $\propto \sqrt{\gamma kT}$

R. A. Duine and H. T. C. Stoof, *Phys. Rev. A*, **65**, (2001);
S. P. Cockburn, *Bose Gases In and Out of Equilibrium within the Stochastic Gross-Pitaevskii Equation*, Ph.D. Thesis (2010)

What is Ψ in SGPE ?

System described by SGPE is divided into **two** parts:

- Few highly occupied low-lying modes which is represented by a **Langevin field** $\Psi(\mathbf{x}, t)$.

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- Langevin field $\Psi = \langle\hat{\Psi}\rangle + \delta\hat{\Psi} \equiv \Phi + \delta\hat{\Psi}$

$\Phi \rightarrow$ Condensate part at $T = 0$
$\delta\hat{\Psi} \rightarrow$ Non-condensate part (thermal fluctuations).

To **extract** the condensate part from the Langevin field, one has to employ **Penrose-Onsager** criterion.

S. P. Cockburn, *Bose Gases In and Out of Equilibrium within the Stochastic Gross-Pitaevskii Equation*, Ph.D. Thesis (2010)

Dynamics of single vortex at $T \neq 0$

- We consider a BEC in *quasi-two dimensional* trap for which $\omega_x = \omega_y \ll \omega_z$. The axial degrees of freedom of the system are frozen.
- We use the scaled two-dimensional equation after integrating out the axial coordinate:

$$(i-\gamma)\frac{\partial\psi}{\partial t} = \left(-\nabla^2_{xy} + 2V_{xy} + 2g_{xy}|\psi|^2 - 2\mu\right)\psi + \frac{2\eta}{1-i\gamma},$$

where $\nabla^2_{xy} = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $V_{xy} = x^2/2 + y^2/2$ and $g_{xy} = \sqrt{\lambda_z/2\pi}g$

Present work

Parameter Set:
We choose $\omega = \omega_x = \omega_y = 2\pi \times 10$ Hz and $\omega_z = 2\pi \times 100$ Hz.
We consider $\approx 1 \times 10^5$-1.5×10^5 atoms of ^{87}Rb.
The *s*-wave scattering length of ^{87}Rb is $99a_0$.
$T_c^{q2d} \approx 44 - 53\ \mathrm{nK}$.
We take dissipation parameter

$$\gamma = \kappa \frac{4mkT}{\pi} \left(\frac{a}{\hbar}\right)^2,$$

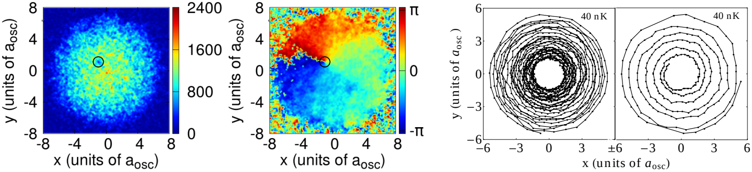
where $\kappa = 3$ (reproduces the growth rate observed in most experiments).
Higher temperature implies greater rate of decay.
We first **generate a single vortex by rotating the condensate** with a frequency $\Omega = 0.1\ \omega_x$ at $T = 30$ nK, 40 nK and 50 nK

Penckwitt *et. al Phys. Rev. Lett.*, **89**, (2002).

Present work

Results:

For a temperature of 40 nK,



The panel on the right shows the trajectory traversed by the vortex for two different values of γ; $\gamma = 7.5 \times 10^{-4}$ (left) and $\gamma = 7.5 \times 10^{-3}$ (right).

Present work

In absence of rotation,

- Vortex slowly spirals out of the condensate. This leads to the decrease in energy of the system, which indicates the energetic instability of the vortex state.

S. Gautam, AR, S. Mukerjee, *Phys. Rev. A*, **89**, 013612 (2014)

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- Vortex slowly spirals out of the condensate. This leads to the decrease in energy of the system, which indicates the energetic instability of the vortex state.
- The lowest mode known as the fundamental Kelvin mode or anomalous mode is stable only if the condensate is rotating at an optimum frequency. In the absence of rotation, the energy of the fundamental Kelvin mode becomes negative, which indicates the energetic instability of this mode.

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- At T = 0 K, when there is no dissipation, an off-center vortex will follow an isoenergetic circular trajectory around the center of the trap .

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- At T = 0 K, when there is no dissipation, an off-center vortex will follow an isoenergetic circular trajectory around the center of the trap .
- At $T \neq 0$, the dissipation slowly reduces the energy of the condensate and hence makes the vortex execute a spiral trajectory towards the edge of the condensate . This leads to a vortex-free state which is energetically stable.

S. Gautam, AR, S. Mukerjee, *Phys. Rev. A*, **89**, 013612 (2014)

Conclusion

- We have studied the dynamics of a single vortex in quasi two-dimensional systems at finite temperatures using the SGPE.
- We find the single vortex tends to decay at finite temperature which is not the case at $T = 0$.

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THANK YOU